

**AFRL-RI-RS-TR-2008-42**  
**Final Technical Report**  
**March 2008**



# **RADIO RESOURCE ALLOCATION AND MULTIUSER DETECTION IN VERY LARGE SCALE WIRELESS MULTIPLE ACCESS NETWORKS**

**Princeton University**

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<b>REPORT DOCUMENTATION PAGE</b>				<i>Form Approved</i> <b>OMB No. 0704-0188</b>	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Service, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.</small>					
<b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</b>					
<b>1. REPORT DATE (DD-MM-YYYY)</b> MAR 2008		<b>2. REPORT TYPE</b> Final		<b>3. DATES COVERED (From - To)</b> Sep 06 – Sep 07	
<b>4. TITLE AND SUBTITLE</b>  RADIO RESOURCE ALLOCATION AND MULTIUSER DETECTION IN VERY LARGE SCALE WIRELESS MULTIPLE ACCESS NETWORKS				<b>5a. CONTRACT NUMBER</b>	
				<b>5b. GRANT NUMBER</b> FA8750-06-1-0252	
				<b>5c. PROGRAM ELEMENT NUMBER</b> 35885G	
<b>6. AUTHOR(S)</b>  H. Vincent Poor, Sharon Betz, Stefano Buzzi, Valeria Massaro, Giacomo Bacci, Marco Luise				<b>5d. PROJECT NUMBER</b> 1038	
				<b>5e. TASK NUMBER</b> BA	
				<b>5f. WORK UNIT NUMBER</b> 04	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> Trustees of Princeton University 1 Nassau Hall Princeton NJ 08544-0001				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  AFRL/RIEC 525 Brooks Rd Rome NY 13441-4505				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b>	
				<b>11. SPONSORING/MONITORING AGENCY REPORT NUMBER</b> AFRL-RI-RS-TR-2008-42	
<b>12. DISTRIBUTION AVAILABILITY STATEMENT</b> APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED. PA# WPAFB 08-1704					
<b>13. SUPPLEMENTARY NOTES</b>					
<b>14. ABSTRACT</b> This study was concerned with the use of game theoretic principles for radio resource management in wireless multiple-access networks. Our progress on this problem is described in detail in publications listed in the technical report, for each of which a link to the full document is provided. These documents examine a number of resource-allocation issues wherein the primary performance criterion is optical energy efficiency. Specific contributions include receiver designs and power allocation in multi-hop (ad hoc) networks, analysis and design of energy-efficient receiver algorithms for ultra-wideband (UWB) systems in rich multi-path environments, extensions of analyses for linear receivers to serial interference cancellers, joint transmitter and receiver optimization for energy efficient network operations, and efficient and adaptive distributed algorithms for achieving optimal designs. Taken as a whole, this work provides guidance for designers of sensor networks and advanced communications networks for achievement of optimal energy efficiency in network operations.					
<b>15. SUBJECT TERMS</b> Very Large Scale Wireless Multiple Access Networks, Radio Resource Allocation and Multi User Detection, Delay Quality of Service in Very Large Networks					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>  UL	<b>18. NUMBER OF PAGES</b>  98	<b>19a. NAME OF RESPONSIBLE PERSON</b> Aaron J. Teter
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			<b>19b. TELEPHONE NUMBER (Include area code)</b> N/A

## **Brief Summary of Findings Under AFRL Cooperative Agreement Grant FA8750-06-1-0252**

This study was concerned with the use of game theoretic principles for radio resource management in wireless multiple-access networks. Our progress on this problem is described in detail in the publications listed below, for each of which a link to the full document is provided. These documents examine a number of resource-allocation issues wherein the primary performance criterion is optimal energy efficiency. Specific contributions include receiver designs and power allocation in multi-hop (ad hoc) networks [1], [2]; analysis and design of energy-efficient receiver algorithms for ultra-wideband (UWB) systems in rich multi-path environments [3] – [5]; extensions of analyses for linear receivers to serial interference cancellers [6]; joint transmitter and receiver optimization for energy efficient network operation [7]; and efficient and adaptive distributed algorithms for achieving optimal designs [8], [9]. Taken as a whole, this work provides guidance for designers of sensor networks and advanced communications networks for achievement of optimal energy efficiency in network operations.

### **Introduction**

In wireless networks, bandwidth and energy are generally scarce. Nodes are often battery-powered so energy efficiency is often a primary consideration in the design and deployment of networks. Furthermore, one node's communication generally causes interference to other nodes, so the nodes' choices are coupled. Game theory provides the tools to study the necessary radio resource management given this coupling between the nodes' actions. The purpose of this work is to provide design guidance for the design of wireless sensor and advanced communication networks to achieve optimal energy efficiency.

### **Methods, Assumptions, and Procedures**

The work is described in detail in nine papers [1-9] attached as Appendices. The primary assumption is that of a spread-spectrum multiple-access communication network with a very large number of nodes; [3-5] consider the case of impulse radio UWB (IR-UWB) systems and [1-2,4,6-9] consider direct-sequence code division multiple access (DS-CDMA). The work analyzes the large system limit: for DS-CDMA, the spreading gain and number of nodes both increase without bound while their ratio (the system load) remains constant, while for IR-UWB, the processing gain (the product of the number of pulses per information symbol and the number of possible pulse positions per frame) and number of nodes both increase without bound, again while their ratio remains constant. As mentioned above, a game-theoretic approach is used, in which the terminals in the network are viewed as "economic" agents competing for radio resources to transmit their messages as efficiently as possible, maximizing their "utility": the number of bits transmitted successfully per joule of energy. The agents attempt to maximize their utilities by modifying their transmit power; papers [1,2,5,6,7] explore the games when the agents choose amongst a set of receivers in addition to choosing their transmit power, in [7] the users also choose their spreading codes, and [4] compares the games with DS-CDMA and IR-UWB. Papers [1-2] examine the multihop case, where all nodes may not be transmitting to the same destination and [2] modifies the utility function to include the power to operate nodes as well as the transmit power. In papers [3-6], the environmental assumptions are loosened to include multipath. The work reported in papers [8-9] develops efficient and adaptive distributed algorithms for achieving optimal designs, including those to the games in papers [1-7].

### **Results and Discussion**

The results are described in detail in the accompanying publications [1-9]. In each of the

presented games, a unique Nash equilibrium is derived and described along with the resulting explicit expression for utility; these results are in the form of the energy efficiency per node (bits-per-joule) as a function of the network parameters. In some cases [1,2], the results are compared to Pareto optimal solutions.

## Conclusions

For detailed conclusions, please see the accompanying publications [1-9] in the appendix. Using game theory, we can derive expressions for the energy efficiency of many systems in terms of only network parameters. Furthermore, these energy efficient solutions for systems can be achieved with simple distributed algorithms that converge to a unique Nash equilibrium or fixed point. The use of multiuser detection can be a major determinant in the energy efficiency of multiuser CDMA systems and spreading code optimization and non-linear reception techniques can bring remarkable performance gains. Finally, with equal spreading factors, DS-CDMA and IR-UWB are nearly equivalent in terms of energy efficiency.

## Appendices: Publications Reporting Results of AFRL Cooperative Agreement Grant FA8750-06-1-0252

1. S. Betz and H. V. Poor, "Energy efficiency in multi-hop CDMA networks: A game theoretic analysis," (invited paper). *Proceedings of the Workshop on Multi-Layer Modelling and Design of Multi-Hop Wireless Networks (MLMD'06)*, Minneapolis, MN, July 12 - 15, 2006. [Posted on ArXiv at <http://www.arxiv.org/abs/cs/0605098>.]
2. S. Betz and H. V. Poor, "Energy efficiency in multi-hop CDMA networks: A game theoretic analysis considering operating costs," *IEEE Transactions on Signal Processing*, undergoing review. [Submitted October 12, 2007.]
3. G. Bacci, M. Luise and H. V. Poor, "Large system analysis of game-theoretic power control in UWB wireless networks with rake receivers," *Proceedings of the Eighth IEEE International Workshop on Signal Processing Advances for Wireless Communications (SPAWC)*, Helsinki, Finland, June 17 - 20, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0704.2017>.]
4. G. Bacci, M. Luise and H.V. Poor, "Performance comparison of energy-efficient power control for CDMA and multiuser UWB networks," *Proceedings of the IST Mobile & Wireless Communications Summit*, Budapest, Hungary, July 1 - 5, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0705.0936>.]
5. G. Bacci, M. Luise and H. V. Poor, "Performance of Rake receivers in IR-UWB networks using energy-efficient power control," *IEEE Transactions on Wireless Communications*, to appear. [Posted on ArXiv at <http://www.arxiv.org/abs/cs/0701034>.]
6. S. Buzzi, V. Massaro and H. V. Poor, "Power control and receiver design for energy efficiency in multipath CDMA channels with bandlimited waveforms," *Proceedings of the 41st Annual Conference on Information Sciences and Systems*, Johns Hopkins University, Baltimore, MD, March 14 - 16, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0704.2383>.]
7. S. Buzzi and H. V. Poor, "Non-cooperative games for spreading code optimization,

power control and receiver design in wireless data networks,” *Proceedings of the 13th European Wireless Conference*, Paris, France, April 1 - 4, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0705.0783>.] [Also to appear in *IEEE Journal on Selected Areas in Communications - Special Issue on Multiuser Detection for Advanced Communication Systems and Networks*

8. S. Buzzi and H. V. Poor, “Power control algorithms for CDMA networks based on large system analysis,” *Proceedings of the 2007 IEEE International Symposium on Information Theory*, Nice, France, June 24 - 29, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0704.2375>.]
9. S. Buzzi, H. V. Poor and D. Saturnino, “A stochastic non-cooperative game for energy efficiency in wireless data networks,” *Proceedings of the 2007 Tyrrhenian International Workshop on Digital Communication*, Naples, Italy, September 9 - 12, 2007. [Posted on ArXiv at <http://www.arxiv.org/abs/0705.3013>.]

# Energy Efficiency in Multi-hop CDMA Networks: A Game Theoretic Analysis\*

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## Abstract

*A game-theoretic analysis is used to study the effects of receiver choice on the energy efficiency of multi-hop networks in which the nodes communicate using Direct-Sequence Code Division Multiple Access (DS-CDMA). A Nash equilibrium of the game in which the network nodes can choose their receivers as well as their transmit powers to maximize the total number of bits they transmit per unit of energy is derived. The energy efficiencies resulting from the use of different linear multiuser receivers in this context are compared, looking at both the non-cooperative game and the Pareto optimal solution. For analytical ease, particular attention is paid to asymptotically large networks. Significant gains in energy efficiency are observed when multiuser receivers, particularly the linear minimum mean-square error (MMSE) receiver, are used instead of conventional matched filter receivers.*

## 1 Introduction

In a wireless multi-hop network, nodes communicate by passing messages for one another; permitting multi-hop communications, rather than requiring one-hop communications, can increase network capacity and allow for a more ad hoc (and thus scalable) system (with little or no centralized control). For these reasons, and because of their potential for commercial, military, and civil applications, wireless multi-hop networks have attracted considerable attention over the past few years. In these networks, energy efficient communication is important because the nodes are typically battery-powered and therefore energy-limited. Work on energy-efficient communication in these multi-hop networks has often focused on routing protocols; this work instead looks at power control and receiver design choices that

can be implemented independently of (and thus in conjunction with) the routing protocol.

One approach that has been very successful in researching energy efficient communications in both cellular and multi-hop networks is the game-theoretic approach described in [1, 2]. Much of the game-theoretic research in multi-hop networks has focused on pricing schemes (e.g. [3, 4]). In this work, we avoid the need for such a pricing scheme by using instead a nodal utility function to capture the energy costs. It further differs from previous research by considering receiver design, as [5] does for cellular networks.

We propose a distributed noncooperative game in which the nodes can choose their transmit power and linear receiver design to maximize the number of bits that they can send per unit of power. After describing the network and internodal communications in Section 2, we derive the Nash equilibrium for this game, as well as for a set of games with set receivers, in Section 3. We then extend the asymptotic work of Tse and Hanly [6] to fit the multihop network structure in Section 4; we apply this in Section 5 to find the Pareto optimal solution in an asymptotically large, SINR-balanced network. Finally we present some numerical results and a conclusion in Sections 6 and 7.

## 2 System Model

Consider a wireless multi-hop network with  $K$  nodes (users) and an established logical topology, where a sequence of connected link-nodes  $l \in L(k)$  forms a route originating from a source  $k$  (with  $k \in L(k)$  by definition). Let  $m(k)$  be the node after node  $k$  in the route for node  $k$ . Assume that all routes that go through a node  $k$  continue through  $m(k)$  so that node  $k$  transmits only to  $m(k)$ . Nodes communicate with each other using DS-CDMA with processing gain  $N$  ( $N$  chips per bit).

The signal received at a node  $m$  (after chip-matched filtering) sampled at the chip rate over one symbol duration

\*This research was supported in part by the U. S. Air Force Research Laboratory and in part by the Defense Advanced Research Projects Agency.

can be expressed as

$$\mathbf{r}^{(m)} = \sum_{k=1}^K \sqrt{p_k} h_k^{(m)} b_k s_k + \mathbf{w}^{(m)} \quad (1)$$

where  $p_k$ ,  $b_k$ , and  $s_k$  are the transmit power, transmitted symbol, and (binary) spreading sequence for node  $k$ ;  $h_k^{(m)}$  is the channel gain between nodes  $k$  and  $m$ ; and  $\mathbf{w}^{(m)}$  is the noise vector which is assumed to be Gaussian with mean 0 and covariance  $\sigma^2 \mathbf{I}$ . (We assume here  $p_m = 0$ .) Assume the spreading sequences are random, i.e.,  $s_k = \frac{1}{\sqrt{N}}[v_1 \dots v_N]^T$ , where the  $v_i$ 's are independent and identically distributed (i.i.d.) random variables taking values  $\{-1, +1\}$  with equal probabilities. Denote the cross-correlations between spreading sequences as

$$\rho_{kj} = s_k^T s_j, \quad (2)$$

noting that  $\rho_{kk} = 1$  for all  $k$ .

Let us represent the linear receiver at the  $m$ th node for the  $k$ th signature sequence by a coefficient vector  $\mathbf{c}_k^{(m)}$ . The output of this receiver can be written as

$$y = \mathbf{c}_k^T \mathbf{r}^{(m)} \quad (3)$$

$$= \sqrt{p_k} h_k^{(m)} b_k c_k^T s_k + \sum_{j \neq k} \sqrt{p_j} h_j^{(m)} b_j c_k^T s_j + c_k^T \mathbf{w}^{(m)}. \quad (4)$$

The signal-to-interference-plus-noise ratio (SINR),  $\gamma_k$ , of the  $k$ th user at the output of receiver  $m(k)$  is

$$\gamma_k = \frac{p_k h_k^{(m(k))2} (c_k^T s_k)^2}{\sigma^2 c_k^T c_k + \sum_{j \neq k} p_j h_j^{(m(k))2} (c_k^T s_j)^2}. \quad (5)$$

Each user has a utility function that is the ratio of its effective throughput to its transmit power, i.e.,

$$u_k = \frac{T_k}{p_k}. \quad (6)$$

Here, the throughput,  $T_k$ , is the net number of information bits sent by  $k$  (generated by  $k$  or any node whose route goes through  $k$ ) and received without error at the intended destination,  $m(k)$ , per unit of time. (We assume that all the congestion control is done in the choice of routing.)

Following the discussion in [5], we will use

$$T_k = \frac{L}{M} R f(\gamma_k) \quad (7)$$

where  $L$  and  $M$  are the number of information bits and the total number of bits in a packet, respectively (without loss of generality assumed here to be the same for all users);  $R$  is the transmission rate, which is the ratio of the bandwidth to the processing gain and is taken for now to be equal for all

users; and  $f(\cdot)$  is an efficiency function that closely approximates the packet success rate. This efficiency function can be any increasing, continuously differentiable, sigmoidal<sup>1</sup> function with  $f(0) = 0$  and  $f(+\infty) = 1$ . See [5] for more discussion of the efficiency function.

Using (7), (6) becomes

$$u_k = \frac{L}{M} R \frac{f(\gamma_k)}{p_k}. \quad (8)$$

When the receiver used is a matched filter (MF) (i.e.  $\mathbf{c}_k^{(m(k))} = s_k$ ), the received SINR is

$$\gamma_k^{\text{MF}} = \frac{p_k h_k^{(m(k))2} (s_k^T s_k)^2}{\sigma^2 s_k^T s_k + \sum_{j \neq k} p_j h_j^{(m(k))2} (s_k^T s_j)^2} \quad (9)$$

$$= \frac{p_k h_k^{(m(k))2}}{\sigma^2 + \sum_{j \neq k} p_j h_j^{(m(k))2} \rho_{kj}^2}. \quad (10)$$

When the receiver is a linear minimum mean-squared error (MMSE) receiver, the filter coefficients and the received SINR are [7]

$$\mathbf{c}_k^{\text{MMSE}} = \frac{\sqrt{p_k} h_k^{(m(k))}}{1 + p_k h_k^{(m(k))2} s_k^T \mathbf{A}_k^{-1} s_k} \mathbf{A}_k^{-1} s_k \quad (11)$$

and

$$\gamma_k^{\text{MMSE}} = p_k h_k^{(m(k))2} s_k^T \mathbf{A}_k^{-1} s_k, \quad (12)$$

where

$$\mathbf{A}_k = \sigma^2 \mathbf{I} + \sum_{j \neq k} p_j h_j^{(m(k))2} s_j s_j^T. \quad (13)$$

When the receiver is a decorrelator<sup>2</sup> (DE) (i.e.  $\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_K] = \mathbf{S}(\mathbf{S}^T \mathbf{S})^{-1}$  where  $\mathbf{S} = [s_1 \dots s_K]$ ), the received SINR is

$$\gamma_k^{\text{DE}} = \frac{p_k h_k^{(m(k))2}}{\sigma^2 \mathbf{c}_k^T \mathbf{c}_k}. \quad (14)$$

For any linear receiver with all nodes' coefficients chosen independently of their transmit powers (including the MF and DE), as well as for the MMSE receiver,

$$\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k}. \quad (15)$$

<sup>1</sup> A continuous increasing function is sigmoidal if there is a point above which the function is concave and below which the function is convex.

<sup>2</sup> Here, we must assume that  $K \leq N$ .



### 3 The Noncooperative Power-Control Game

Let  $\mathcal{G} = [\mathcal{K}, \{A_k\}, \{u_k\}]$  denote the noncooperative game where  $\mathcal{K} = \{1, \dots, K\}$  and  $A_k = [0, P_{\max}] \times \mathbb{R}^N$  is the strategy set for the  $k$ th user. Here,  $P_{\max}$  is the maximum allowed power for transmission. Each strategy in  $A_k$  can be written as  $\mathbf{a}_k = (p_k, \mathbf{c}_k)$ , where  $p_k$  and  $\mathbf{c}_k$  are the transmit power and the receiver filter coefficients, respectively, of user  $k$ . Then the resulting noncooperative game can be expressed as the maximization problem for  $k = 1, \dots, K$ :

$$\max_{\mathbf{a}_k} u_k = \frac{LR}{M} \max_{p_k, \mathbf{c}_k} \frac{f(\gamma_k(p_k, \mathbf{c}_k))}{p_k}, \quad (16)$$

where  $\gamma_k$  is expressed explicitly as a function of  $p_k$  and  $\mathbf{c}_k$ .

This is similar to the noncooperative power-control game in [5]; here, however, the channel gains are between pairs of nodes rather than between a node and the base-station.

Since the choice of receiver is independent of the transmit power and  $f(\cdot)$  is an increasing function, the analysis of [5] applies, so the maximization from (16) becomes:

$$\max_{p_k, \mathbf{c}_k} \frac{f(\gamma_k(p_k, \mathbf{c}_k))}{p_k} = \max_{p_k} \frac{f(\max_{\mathbf{c}_k} \gamma_k(p_k, \mathbf{c}_k))}{p_k}. \quad (17)$$

Note that the MMSE receiver achieves the maximum SINR amongst all linear receivers, so that if a Nash equilibrium exists, at that equilibrium all receivers must be MMSE receivers. Then the maximization problem becomes

$$\max_{p_k} \frac{f(\gamma_k^{\text{MMSE}}(p_k))}{p_k}. \quad (18)$$

Let  $\mathcal{G}_C = [\mathcal{K}, \{[0, P_{\max}]\}, \{u_k\}]$  denote the noncooperative game that differs from  $\mathcal{G}$  in that users cannot choose their linear receivers but are forced to use the receive filter coefficients  $[\mathbf{c}_1 \dots \mathbf{c}_K] = \mathbf{C}$  (which may be a function of the powers,  $\mathbf{P}$ ). The resulting noncooperative game can be expressed as the following maximization problem for  $k = 1, \dots, K$ :

$$\max_{\mathbf{a}_k} u_k = \max_{p_k} u_k(p_k, \mathbf{c}_k) = \frac{LR}{M} \max_{p_k} \frac{f(\gamma_k^{\mathbf{C}}(p_k))}{p_k} \quad (19)$$

where  $\gamma_k^{\mathbf{C}}$  is expressed explicitly as a function of  $p_k$ . Then the maximization problem in (18) is one of the games  $\mathcal{G}_C$  when  $\mathbf{C}$  is chosen to be the MMSE receivers.

For any  $\mathbf{C}$  matrix (or  $\mathbf{C}(\mathbf{P})$  for which (15) holds), the utility function for each user is maximized when

$$p_k = \min\{P_k, p_k^*\} \quad (20)$$

where  $p_k^*$  is the unique positive number that satisfies

$$f(\gamma_k^{\mathbf{C}}(p_k^*)) = \gamma_k^{\mathbf{C}}(p_k^*) f'(\gamma_k^{\mathbf{C}}(p_k^*)). \quad (21)$$

As long as the users all have the same efficiency function,

$$\gamma_1^{\mathbf{C}}(p_1^*) = \dots = \gamma_K^{\mathbf{C}}(p_K^*) = \gamma^* \quad (22)$$

where  $\gamma^*$  is the unique positive number that satisfies

$$f(\gamma^*) = \gamma^* f'(\gamma^*). \quad (23)$$

Finally, since  $\frac{f(\gamma_k)}{p_k}$  is quasi-concave<sup>3</sup> in  $p_k$ , we can use the result cited in [2, Appendix I]:  $\mathcal{G}_C$  has a Nash equilibrium and, as is the case in [5], it is unique. At this equilibrium, unless there is a node  $k$  with  $p_k^* > P_k$ , the powers are such that the nodes are SINR-balanced (i.e. (22) holds).

Returning to the game  $\mathcal{G}$ , a similar result holds: there exists a unique equilibrium where all receivers are MMSE detectors and, if the power limit is high enough, the powers are SINR-balanced.

### 4 Asymptotically Large Systems: Extending the Tse-Hanly Equations to Multi-Hop Networks

Assume that the channel gains are independent. That is, in the asymptotic regime when  $N, K \rightarrow \infty$  while  $K/N = \beta$ , the interferers' channel gains,  $h_k^{(m)^2}$  for all  $m \neq k, m(k)$ , are iid realizations of the random variable  $G$  with pdf  $f_G$ , and the primary channel gains,  $h_k^{(m(k))^2}$  for all  $k$ , are iid realizations of the random variable  $H$  with pdf  $f_H$  (where  $f_H(h) = 0 \forall h \leq 0$ ). Let  $q = \mathbb{P}\{m(j) = m(k)\}$  for all  $j \neq k$ .

We can apply results from [6] to analyze the nodes' SINRs. Then we find a probability density function for  $p$  such that in an asymptotically large system where all nodes have powers distributed by this function, with probability one all nodes have SINR of at least  $\gamma$  for some  $\gamma$ . If this distribution is not unique, we choose the one that minimizes the nodes' powers. For simplicity, and since we are considering the asymptotic regime, we assume that the distribution of  $p_k$  is independent of all channel gains except for  $h_k^{(m(k))^2}$ . For convenience of notation, let  $f_{p,H}(\cdot, \cdot) = f_{p_k, h_k^{(m(k))^2}}(\cdot, \cdot)$ , and note  $\int_0^\infty f(p, h) dp = f_H(h)$  for all  $h$ . Then the joint density of  $p_k, h_k^{(m(k))^2}$ , and  $h_k^{(m(j))^2}$  for  $j \neq k$  is

$$\begin{aligned} f_{p_k, h_k^{(m(k))^2}, h_k^{(m(j))^2}}(p, h, g) &= f_{p,H}(p, h) \delta(g - h) q \\ &\quad + f_{p,H}(p, h) f_G(g) (1 - q). \end{aligned} \quad (24)$$

Applying the results from [6], when the receiver at node  $k$  is a matched filter, decorrelator, or MMSE receiver, the random SINR at the receiver converges in probability as

<sup>3</sup>A function is quasi-concave if there exists a point below which the function is nondecreasing and above which the function is non-increasing.

$N, K \rightarrow \infty$  while  $K/N = \beta$ . These asymptotic SINRs are uniquely described by the equations (where  $j \neq k$ ):

$$\gamma^{\text{MF}} = \frac{p_k h_k^{(m(k))^2}}{\sigma^2 + \beta \mathbb{E} \left[ p_j h_j^{(m(j))^2} \right]} \quad (25)$$

$$\gamma^{\text{DE}} = \begin{cases} \frac{p_k h_k^{(m(k))^2 (1-\beta)}}{\sigma^2}, & \alpha < 1; \\ 0, & \alpha \geq 1. \end{cases} \quad (26)$$

and

$$\gamma^{\text{MMSE}} = \frac{p_k h_k^{(m(k))^2}}{\sigma^2 + \beta \int_0^\infty dp \int_0^\infty dg f_P(p) f_G(g) I(pg, p_k h_k^{(m(k))^2}, \gamma^{\text{MMSE}})}, \quad (27)$$

where  $I(a, b, c) = \frac{ab}{b+ac}$ .

If the nodes choose their transmit powers so that the SINRs are balanced, the following theorem determines what SINRs are achievable at all receivers as well as the minimum transmit powers to achieve any achievable SINR when the nodes use the MMSE receiver, under the assumptions listed above.

**Theorem 4.1.** *A necessary and sufficient condition for an SINR,  $\gamma$ , to be achievable is for*

$$\beta \gamma q \frac{1}{1+\gamma} + \beta \gamma (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right] < 1. \quad (28)$$

When (28) holds, each user can achieve the desired SINR,  $\gamma$ , and the minimum power solution to do so is to assign each node,  $k$ , transmit power

$$p_k = P_{\text{MMSE}} \left( h_k^{(m(k))^2}, \gamma \right) \quad (29)$$

$$= \frac{1}{h_k^{(m(k))^2}} \cdot \frac{\gamma \sigma^2}{1 - \beta \gamma q \frac{1}{1+\gamma} - \beta \gamma (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right]}. \quad (30)$$

#### 4.1 Proof of Theorem 4.1

We start with a lemma that is a straightforward consequence of the definition of  $I(a, b, c)$ .

**Lemma 4.2.** *For all positive real numbers  $a_0, a, b, c, a_0 \leq a$  if and only if  $I(a_0, b, c) \leq I(a, b, c)$ .*

Then the proof follows.

*Proof.* To show necessity, assume that there is a pdf  $f$  with  $\int_0^\infty f(p, h) dp = f_H(h)$  for all  $h$ , such that in an asymptotically large system where all nodes have powers and primary channel gains distributed by  $f$ , with probability one

all nodes have SINR when using an MMSE receiver of at least  $\gamma$  for some set  $\gamma$ . Let  $Q = \inf\{ph : f(p, h) > 0\}$ . Then

$$\begin{aligned} \frac{Q}{\gamma} &\geq \sigma^2 + \beta \int_0^\infty dg \int_0^\infty dp \int_0^\infty dh f_{p_k, h_k^{(m(k))^2}, h_k^{(m(j))^2}}(p, h, g) I(pg, Q, \gamma) \\ &= \sigma^2 + \beta q \int_0^\infty dp \int_0^\infty dh f_{p, H}(p, h) I(ph, Q, \gamma) \\ &\quad + \beta(1-q) \int_0^\infty dg \int_0^\infty dp \int_0^\infty dh f_{p, H}(p, h) f_G(g) I(ph \frac{g}{h}, Q, \gamma) \\ &\geq \sigma^2 + \beta q \int_0^\infty dp \int_0^\infty dh f_{p, H}(p, h) I(Q, Q, \gamma) \\ &\quad + \beta(1-q) \int_0^\infty dg \int_0^\infty dp \int_0^\infty dh f_{p, H}(p, h) f_G(g) I(Q \frac{g}{h}, Q, \gamma) \\ &= \sigma^2 + \beta q \frac{Q}{1+\gamma} + \beta(1-q) \int_0^\infty dg \int_0^\infty dh f_H(h) f_G(g) \frac{gQ}{h+\gamma g} \\ &= \sigma^2 + Q \beta q \frac{1}{1+\gamma} + Q \beta (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right]. \end{aligned} \quad (31)$$

This implies that

$$Q \left( 1 - \beta \gamma q \frac{1}{1+\gamma} - \beta \gamma (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right] \right) \geq \gamma \sigma^2 > 0, \quad (32)$$

so  $\beta \gamma q \frac{1}{1+\gamma} + \beta \gamma (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right] < 1$ , proving necessity.

When (28) holds, it is easy to show that  $P_{\text{MMSE}}(h, \gamma)$  is positive for all primary channel gains,  $h$ . It is also straightforward to show that if each node,  $k$ , uses transmit power  $P_{\text{MF}}(h_k^{(m(k))^2}, \gamma)$ , all nodes will achieve the SINR requirement,  $\gamma$ , finishing the proof of sufficiency.

Finally, consider any other joint distribution of powers and primary channel gains whose marginal distribution for  $H$  is  $f_H$ , and let  $Q^*$  be the minimal received power in this distribution. Then by exactly the same argument as was used in the proof of necessity,

$$Q^* \geq \frac{\gamma \sigma^2}{1 - \beta \gamma q \frac{1}{1+\gamma} - \beta \gamma (1-q) \mathbb{E} \left[ \frac{G}{H+\gamma G} \right]} \quad (33)$$

$$= h P_{\text{MMSE}}(h, \gamma), \quad \forall h > 0. \quad (34)$$

This means that assigning powers according to  $P_{\text{MMSE}}$  does indeed give the minimal power solution.  $\square$

## 5 A Global Optimization Problem

A useful global optimization problem is

$$\max \sum_{k=1}^K \alpha_k u_k = \frac{L}{M} R \max \sum_{k=1}^K \frac{\alpha_k f(\gamma_k)}{p_k}, \quad (35)$$

where the  $\alpha_k$ 's are set weighting variables. This problem is equivalent to finding a Pareto-optimal solution of the game.

According to [5], even in the special case of a cellular system where  $L(k) = \{k\}$  for all nodes  $k = 1, 2, \dots, K$  and all nodes are transmitting to the base-station, "Pareto-optimal solutions are, in general, difficult to obtain." For simplicity, we restrict the problem by requiring that the solution is "fair": all nodes have equal receiver output SINRs (i.e. SINR-balancing), so  $\gamma = \gamma_1 = \gamma_2 = \dots \gamma_K$ .

With this assumption, (35) becomes

$$\frac{L}{M} R \max_{\gamma} f(\gamma) \sum_{k=1}^K \frac{\alpha_k}{p_k}. \quad (36)$$

For the matched filter, we can apply (5) with  $m = m(k)$  to see that the users' SINRs are equal if and only if

$$\left( B + \left( \frac{1}{\gamma} + 1 \right) D \right) \mathbf{p}(\gamma) = \sigma^2 \mathbf{1} \quad (37)$$

where  $B$  is a  $K$  by  $K$  matrix with entries  $B_{kj} = -h_j^{(m(k))2} \rho_{kj}^2$ ,  $D$  is  $K$  by  $K$  diagonal matrix with diagonal entries  $D_{kk} = h_k^{(m(k))2}$ , and  $\mathbf{1}$  is a vector of  $K$  ones.

The SINR that maximizes (36) is the  $\gamma$  that satisfies

$$0 = \frac{\partial}{\partial \gamma} \left[ f(\gamma) \sum_{k=1}^K \frac{\alpha_k}{p_k(\gamma)} \right] \quad (38)$$

$$= \frac{\partial}{\partial \gamma} [f(\gamma)] \sum_{k=1}^K \frac{\alpha_k}{p_k(\gamma)} - f(\gamma) \sum_{k=1}^K \frac{\alpha_k}{p_k^2(\gamma)} \frac{\partial}{\partial \gamma} [p_k(\gamma)], \quad (39)$$

where  $p_k(\gamma)$  and  $\frac{\partial}{\partial \gamma} [p_k(\gamma)]$  are the  $k$ th elements of

$$\mathbf{p}(\gamma) = \sigma^2 \left( B + \left( \frac{1}{\gamma} + 1 \right) D \right)^{-1} \mathbf{1} \quad (40)$$

and

$$\frac{\partial}{\partial \gamma} [\mathbf{p}(\gamma)] = \sigma^2 (\gamma B + (1 + \gamma) D)^{-1} D (\gamma B + (1 + \gamma) D)^{-1} \mathbf{1}. \quad (41)$$

For the decorrelator, it is easy to show that the non-cooperative results are equal to the globally optimal results, since the users' achieved SINRs are independent of all the powers of all interferers.

Finally, for the MMSE receiver, we can apply the results from Section 4. In a large system, if all users choose their transmit powers based on the values of  $h_k^{(m(j))}$  for  $m(j) \neq m(k)$  only through the average of these interference gains and if we use the assumptions of Section 4, the SINR is approximated by

$$\gamma_k^{\text{MMSE}} \approx \frac{p_k h_k^{(m(k))2}}{\sigma^2 + \frac{1}{N} \sum_{j \neq k} I(p_j h_j^{(m(k))2}, p_k h_k^{(m(k))2}, \gamma_k^{\text{MMSE}})}. \quad (42)$$

Any  $\gamma_k$  which satisfies  $\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k}$  is a solution to (42).

Then the power for user  $k$  to achieve the SINR  $\gamma^*$  is

$$p_k^{\text{MMSE}} = \frac{1}{h_k^{(m(k))2}} \frac{\gamma^* \sigma^2}{1 - \beta \gamma^* \left( q \frac{1}{1 + \gamma^*} + (1 - q) \zeta(\gamma^*) \right)}, \quad (43)$$

where  $\zeta(\gamma)$  is the mean value of  $\frac{G}{H + \gamma G}$  in the network. Equal received SINRs amongst the users is achieved with minimum power consumption when  $p_k h_k^{(m(k))2} = \kappa(\gamma)$  is constant for all  $k$  and

$$\kappa(\gamma) = \frac{\gamma \sigma^2}{1 - \beta \gamma \left( q \frac{1}{1 + \gamma} + (1 - q) \zeta(\gamma) \right)}. \quad (44)$$

Then, (36) can be expressed as

$$\frac{L}{M} R \left( \sum_{k=1}^K \alpha_k h_k^{(m(k))2} \right) \max_{\gamma} \frac{f(\gamma)}{\kappa(\gamma)}. \quad (45)$$

The solution to  $\max_{\gamma} \frac{f(\gamma)}{\kappa(\gamma)}$  must satisfy  $\frac{\partial}{\partial \gamma} \left( \frac{f(\gamma)}{\kappa(\gamma)} \right) = 0$ . Combining this with (44) gives the equation that must be satisfied by the solution to the maximization problem in (45):

$$f(\gamma) = \gamma f'(\gamma) \left( 1 - \frac{\frac{\beta q \gamma}{(1 + \gamma)^2} + \beta(1 - q) \gamma \zeta(\gamma)}{1 - \frac{\beta q \gamma^2}{(1 + \gamma)^2} - \beta(1 - q) \gamma \zeta(\gamma)} \right). \quad (46)$$

If  $\zeta(\gamma) \ll 1$ , then the equation is approximately the same as in the cellular case [5] with  $K/N \rightarrow \beta q$ . Then, the ability to use multiple hops to communicate, and therefore reduce transmit power, has similar results to reducing the system load; furthermore, for a large range of values of  $\beta q$ , the MMSE target SINRs for the noncooperative game and for the Pareto-optimal solution are close.

## 6 Numerical Results

Consider a multi-hop network with  $K = 100$  nodes distributed randomly in a square 500 meters by 500 meters surrounding an access point in the center. We use a simple routing scheme where all nodes transmit to the closest node that is closer to the access point (or the access point if that is closest). We assume that each packet contains 100 bits of data and no overhead ( $L = M = 100$ ); the transmission rate is  $R = 100$  kb/s; the thermal noise power is  $\sigma^2 = 5 \times 10^{-16}$  Watts; the channel gains are distributed with a Rayleigh distribution with mean  $0.3d^{-2}$ , where  $d$  is the distance between the transmitter and receiver; and the processing gain is  $N$ . We use the same efficiency function as [5], namely  $f(\gamma) = (1 - e^{-\gamma})^M$ .

Table 1 shows the average utility for four representative sets of randomly chosen spreading sequences, one for each

of  $N = 50, 100, 200$ , and  $300$ , comparing the mean utility under the various power choice method discussed above. Table 2 shows the target SINRs for the socially optimal results displayed in Table 1.

	MF	DE	MMSE
$N = 50$			
non-coop.	0		$1.198 \times 10^{10}$
soc. opt.	$2.025 \times 10^{-14}$		$1.199 \times 10^{10}$
$N = 100$			
non-coop.	0	$1.095 \times 10^8$	$1.417 \times 10^{10}$
soc. opt.	$1.050 \times 10^{-4}$	$1.095 \times 10^8$	$1.417 \times 10^{10}$
$N = 200$			
non-coop.	0	$7.459 \times 10^9$	$1.476 \times 10^{10}$
soc. opt.	$2.512 \times 10^{-10}$	$7.459 \times 10^9$	$1.476 \times 10^{10}$
$N = 300$			
non-coop.	0.2056	$1.001 \times 10^{10}$	$1.493 \times 10^{10}$
soc. opt.	$1.351 \times 10^9$	$1.001 \times 10^{10}$	$1.493 \times 10^{10}$

**Table 1. Mean utilities for four representative sets of spreading sequences.**

$N$	MF	DE	MMSE
50	0.87		6.39
100	1.31	6.47	6.43
200	0.99	6.47	6.45
300	5.03	6.47	6.46

**Table 2. Socially optimal SINRs for the same four representative sets of spreading sequences.**

The socially optimally implemented MF receiver performs poorly in heavily-loaded systems, while the non-cooperative implementation fails to achieve non-zero utility except in the case with the lightest load. Even in the case where  $\beta = 1/3$ , the mean utility for the socially optimal MF receiver is less than a tenth of the MMSE receiver's mean utility. Using the DE receiver (for which we require that  $K \geq N$ ), as was noted in Section 5, there is no difference between the non-cooperative and socially optimal results: both cases have the same target SINR and thus the same mean utility. For the MMSE receiver, this difference between the mean utility in the non-cooperative and socially optimal implementations is very small. Finally, the DE and MMSE receivers both significantly outperform the MF receiver in all four of these cases. There is, however, a price to pay in using the better-performing receivers: these receivers require more information at every node as well as significantly more computation. These issues will be addressed further in later research.

## 7 Conclusion

We have analyzed the cross-layer issue of energy-efficient communication in multi-hop networks using a game theoretic method. Focusing on linear receivers, we have derived the transmit power levels that results in a Nash equilibrium for multiple receiver designs, showing that at this equilibrium the users are SINR-balanced. We then generalized the important asymptotic work of Tse and Hanly to allow for the case where users and their interferers may be transmitting to different locations, keeping the cellular example as a special case. We applied these asymptotic results, as well as exact results for the MF and DE receivers, to find the equations for the SINR-balanced Pareto-optimal solution. We showed that the MMSE receiver is the optimal receiver and that in many cases the non-cooperative MMSE receiver results are quite close to the socially optimal results.

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# Energy Efficiency in Multi-hop CDMA Networks: A Game Theoretic Analysis Considering Operating Costs

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## Abstract

A game-theoretic analysis is used to study the effects of receiver choice and transmit power on the energy efficiency of multi-hop networks in which the nodes communicate using Direct-Sequence Code Division Multiple Access (DS-CDMA). A Nash equilibrium of the game in which the network nodes can choose their receivers as well as their transmit powers to maximize the total number of bits they transmit per unit of energy spent (including both transmit and operating energy) is derived. The energy efficiencies resulting from the use of different linear multiuser receivers in this context are compared for the non-cooperative game. Significant gains in energy efficiency are observed when multiuser receivers, particularly the linear minimum mean-square error (MMSE) receiver, are used instead of conventional matched filter receivers.

## Index Terms

Energy efficiency, game theory, utility function, Nash equilibrium, cross-layer design.

## I. INTRODUCTION

In a wireless multi-hop network, nodes communicate by passing messages for one another; permitting multi-hop communications, rather than requiring one-hop communications, can increase network capacity and allow for a more ad hoc (and thus scalable) system (with little or no centralized control). For

This work was supported in part by grants from the Air Force Research Laboratory, the Defense Advanced Research Projects Agency, the National Science Foundation, and in part by a graduate fellowship from the Intel Corporation.

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these reasons, and because of their potential for commercial, military, and civil applications, wireless multi-hop networks have attracted considerable attention over the past few years. In these networks, energy efficient communication is important because the nodes are typically battery-powered and therefore energy-limited. Work on energy-efficient communication in these multi-hop networks has often focused on routing protocols; this work instead looks at power control and receiver design choices that can be implemented independently of (and thus in conjunction with) the routing protocol.

One approach that has been very successful in researching energy efficient communications in both cellular and multi-hop networks is the game-theoretic approach described in [1], [2]. Much of the game-theoretic research in multi-hop networks has focused on pricing schemes (e.g. [3], [4]). In this work, we avoid the need for such a pricing scheme by using instead a nodal utility function to capture the energy costs. It further differs from previous research by considering receiver design, as [5] does for cellular networks. This work further differs from existing research, including [6], through an extension of the utility function that considers the total energy costs, not just the transmit energy.

We propose a distributed noncooperative game in which the nodes can choose their transmit power and linear receiver design to maximize the number of bits that they can send per unit of power. After describing the network and internodal communications in Section II, we describe the Nash equilibrium for this game, as well as for a set of games with set receivers, in Section III. We present numerical results and a conclusion in Sections IV and V.

## II. SYSTEM MODEL

Consider a wireless multi-hop network with  $K$  nodes (users) and an established logical topology, where a sequence of connected link-nodes  $l \in L(k)$  forms a route originating from a source  $k$  (with  $k \in L(k)$  by definition). Let  $m(k)$  be the node after node  $k$  in the route for node  $k$ . Assume that all routes that go through a node  $k$  continue through  $m(k)$  so that node  $k$  transmits only to  $m(k)$ . Nodes communicate with each other using DS-CDMA with processing gain  $N$  ( $N$  chips per bit).

The signal received at a node  $m$  (after chip-matched filtering) sampled at the chip rate over one symbol duration can be expressed as

$$\mathbf{r}^{(m)} = \sum_{k=1}^K \sqrt{p_k} h_k^{(m)} b_k s_k + \mathbf{w}^{(m)} \quad (1)$$

where  $p_k$ ,  $b_k$ , and  $s_k$  are the transmit power, transmitted symbol, and (binary) spreading sequence for node  $k$ ;  $h_k^{(m)}$  is the channel gain between nodes  $k$  and  $m$ ; and  $\mathbf{w}^{(m)}$  is the noise vector which is assumed to be Gaussian with mean  $\mathbf{0}$  and covariance  $\sigma^2 \mathbf{I}$ . (We assume here  $p_m = 0$ .) Assume the spreading sequences

are random, i.e.,  $\mathbf{s}_k = \frac{1}{\sqrt{N}}[v_1 \dots v_N]^T$ , where the  $v_i$ 's are independent and identically distributed (i.i.d.) random variables taking values  $\{-1, +1\}$  with equal probabilities. Denote the cross-correlations between spreading sequences as

$$\rho_{kj} = \mathbf{s}_k^T \mathbf{s}_j, \quad (2)$$

noting that  $\rho_{kk} = 1$  for all  $k$ .

Let the vector  $\mathbf{c}_k^{(m)}$  represent the linear receiver at the  $m$ th node for the  $k$ th signature sequence. The output of this receiver can be written as

$$y = \mathbf{c}_k^T \mathbf{r}^{(m)} \quad (3)$$

$$= \sqrt{p_k} h_k^{(m)} b_k \mathbf{c}_k^T \mathbf{s}_k + \sum_{j \neq k} \sqrt{p_j} h_j^{(m)} b_j \mathbf{c}_k^T \mathbf{s}_j + \mathbf{c}_k^T \mathbf{w}^{(m)}. \quad (4)$$

The signal-to-interference-plus-noise ratio (SINR),  $\gamma_k$ , of the  $k$ th user at the output of receiver  $m(k)$  is

$$\gamma_k = \frac{p_k h_k^{(m(k))2} (\mathbf{c}_k^T \mathbf{s}_k)^2}{\sigma^2 \mathbf{c}_k^T \mathbf{c}_k + \sum_{j \neq k} p_j h_j^{(m(k))2} (\mathbf{c}_k^T \mathbf{s}_j)^2}. \quad (5)$$

Each user has a utility function that is the ratio of its effective throughput to its expended transmit and computation power, i.e.,

$$u_k = \frac{T_k}{p_k + q_k}. \quad (6)$$

Here, the throughput,  $T_k$ , is the net number of information bits sent by  $k$  (generated by  $k$  or any node whose route goes through  $k$ ) and received without error at the intended destination,  $m(k)$ , per unit of time and  $q_k$  is the power expended by the node to implement the receiver. (We assume that all the congestion control is done in the choice of routing.)

Following the discussion in [5], we will use

$$T_k = \frac{L}{M} R f(\gamma_k) \quad (7)$$

where  $L$  and  $M$  are the number of information bits and the total number of bits in a packet, respectively (without loss of generality assumed here to be the same for all users);  $R$  is the transmission rate, which is the ratio of the bandwidth to the processing gain and is taken for now to be equal for all users; and  $f(\cdot)$  is an efficiency function that closely approximates the packet success rate. This efficiency function can be any increasing, continuously differentiable, sigmoidal<sup>1</sup> function with  $f(0) = 0$  and  $f(+\infty) = 1$ . Let its

<sup>1</sup>A continuous increasing function is sigmoidal if there is an inflection point above which the function is concave and below which the function is convex.

first derivative be denoted as  $f'(\gamma) = \frac{\partial f(\gamma)}{\partial \gamma}$  and let  $\gamma_0$  be its inflection point. See [5] for more discussion of the efficiency function.

Using (7), (6) becomes

$$u_k = \frac{L}{M} R \frac{f(\gamma_k)}{p_k + q_k}. \quad (8)$$

When the receiver used is a matched filter (MF) (i.e.  $\mathbf{c}_k^{(m(k))} = \mathbf{s}_k$ ), the received SINR is

$$\gamma_k^{\text{MF}} = \frac{p_k h_k^{m(k)^2} (\mathbf{s}_k^T \mathbf{s}_k)^2}{\sigma^2 \mathbf{s}_k^T \mathbf{s}_k + \sum_{j \neq k} p_j h_j^{m(k)^2} (\mathbf{s}_k^T \mathbf{s}_j)^2} \quad (9)$$

$$= \frac{p_k h_k^{m(k)^2}}{\sigma^2 + \sum_{j \neq k} p_j h_j^{m(k)^2} \rho_{kj}^2}. \quad (10)$$

When the receiver is a linear minimum mean-squared error (MMSE) receiver, the filter coefficients and the received SINR are [7]

$$\mathbf{c}_k^{\text{MMSE}} = \frac{\sqrt{p_k} h_k^{m(k)}}{1 + p_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \mathbf{A}_k^{-1} \mathbf{s}_k \quad (11)$$

and

$$\gamma_k^{\text{MMSE}} = p_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k, \quad (12)$$

where

$$\mathbf{A}_k = \sigma^2 \mathbf{I} + \sum_{j \neq k} p_j h_j^{m(k)^2} \mathbf{s}_j \mathbf{s}_j^T. \quad (13)$$

When the receiver is a decorrelator<sup>2</sup> (DE) (i.e.  $\mathbf{C} = [\mathbf{c}_1 \cdots \mathbf{c}_K] = \mathbf{S}(\mathbf{S}^T \mathbf{S})^{-1}$  where  $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_K]$ ), the received SINR is

$$\gamma_k^{\text{DE}} = \frac{p_k h_k^{m(k)^2}}{\sigma^2 \mathbf{c}_k^T \mathbf{c}_k}. \quad (14)$$

For any linear receiver with all nodes' coefficients chosen independently of their transmit powers (including the MF and DE), as well as for the MMSE receiver, the SINR for user  $k$  is the product of user  $k$ 's power and a factor that is independent of user  $k$ 's power:

$$\gamma_k(p_k, \mathbf{p}_{-k}) = p_k g_k(\mathbf{p}_{-k}), \quad (15)$$

<sup>2</sup>Here, we must assume that  $K \leq N$ .



where  $\mathbf{p}_{-k}$  is a vector of the powers of all users except for user  $k$  and  $g_k$  is a function that depends on the receiver type, the channel gains,  $q_k$ , and the users' spreading sequences. This means that

$$\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k} = g_k(\mathbf{p}_{-k}), \quad (16)$$

so  $\gamma_k$  is strictly increasing in  $p_k$ . Thus, for a fixed receiver type and fixed powers for the other users, there is a one-to-one relationship between the power of user  $k$  and its SINR. Let  $p_0(\mathbf{p}_{-k}) = \frac{\gamma_0}{g_k(\mathbf{p}_{-k})}$  be the unique positive number for which  $\gamma_k(p_0(\mathbf{p}_{-k}), \mathbf{p}_{-k}) = \gamma_0$ , where, as before,  $\gamma_0$  is the inflection point of the efficiency function  $f(\gamma)$ .

### III. THE NONCOOPERATIVE POWER-CONTROL GAME

Let  $\mathcal{G} = [\mathcal{K}, \{A_k\}, \{u_k\}]$  denote the noncooperative game where  $\mathcal{K} = \{1, \dots, K\}$  and  $A_k = [0, P_{\max}] \times \mathcal{R}$  is the strategy set for the  $k$ th user. Here,  $P_{\max}$  is the maximum allowed power for transmission and  $\mathcal{R}$  is the set of allowable receivers, for now restricted to the MF, DE, and MMSE receivers. Each strategy in  $A_k$  can be written as  $\mathbf{a}_k = (p_k, r_k)$ , where  $p_k$  and  $r_k$  are the transmit power and the receiver type, respectively, of user  $k$ . Then the resulting noncooperative game can be expressed as the maximization problem for  $k = 1, \dots, K$ :

$$\max_{\mathbf{a}_k} u_k = \max_{p_k, r_k} u_k(p_k, r_k) \quad (17)$$

$$= \frac{L}{M} R \max_{p_k, r_k} \frac{f(\gamma_k^{r_k}(p_k, \mathbf{p}_{-k}))}{p_k + q_k^{r_k}} \quad (18)$$

$$= \frac{L}{M} R \max_{r_k} \left( \max_{p_k} \frac{f(\gamma_k^{r_k}(p_k, \mathbf{p}_{-k}))}{p_k + q_k^{r_k}} \right), \quad (19)$$

where  $\gamma_k$  and  $q_k$  are expressed explicitly as functions of the transmit power and receiver type.

For each of the receivers,  $r$ , in  $\mathcal{R}$ , let  $\mathcal{G}_r = [\mathcal{K}, \{[0, P_{\max}]\}, \{u_k\}]$  denote the noncooperative game that differs from  $\mathcal{G}$  in that users cannot choose their linear receivers but are forced to use the receiver  $r$ . The resulting noncooperative game can be expressed as the following maximization problem for  $k = 1, \dots, K$ :

$$\max_{p_k} u_k(p_k, r) = \frac{LR}{M} \max_{p_k} \frac{f(\gamma_k^r(p_k, \mathbf{p}_{-k}))}{p_k + q_k}. \quad (20)$$

Given a certain system and fixed powers for all other users, there is a unique optimal power level for any user. The next three lemmas show this and describe that power level.

**Lemma III.1.** *For any user  $k$ , fixed non-negative scalars  $g$  and  $q_k$ , and linear receiver discussed above, there is a unique positive scalar  $\widehat{p}_k(g)$  that satisfies*

$$\left. \frac{\partial f(pg)}{\partial p} \right|_{p=\widehat{p}_k(g)} = 0 \quad (21)$$

and it occurs in the concave region of the efficiency function:  $g\widehat{p}_k(g) > \gamma_0 \forall g \in \mathbb{R}_+$ .

*Proof:* We prove the statement for any  $k \in \{1, 2, \dots, K\}$ . First, choose any  $p \leq \frac{\gamma_0}{g}$  and let  $\gamma = pg \in [0, \gamma_0]$ , which implies that  $f'(\gamma) > \frac{f(\gamma)}{\gamma}$ . Then

$$\left. \frac{\partial}{\partial p_k} \frac{f(p_k g)}{p_k + q_k} \right|_{p_k=p} = \frac{\gamma f'(\gamma)}{p(p+q_k)} - \frac{f(\gamma)}{(p+q_k)^2} \quad (22)$$

$$> \frac{f(\gamma)}{p(p+q_k)} - \frac{f(\gamma)}{(p+q_k)^2} \quad (23)$$

$$= \frac{q_k f(\gamma)}{p(p+q_k)^2} \quad (24)$$

$$\geq 0. \quad (25)$$

This means that no value of  $p \leq \frac{f(\gamma)}{g}$  satisfies (21).

Next, let  $b(\gamma) = f(\gamma) - \gamma f'(\gamma)$ , so

$$\frac{\partial}{\partial p_k} \frac{f(p_k g)}{p_k + q_k} = \frac{(p_k + q_k) f'(p_k g) g - f(p_k g)}{(p_k + q_k)^2} \quad (26)$$

$$= \frac{q_k g f'(p_k g) - b(p_k g)}{(p_k + q_k)^2} \quad (27)$$

Since  $\lim_{\gamma \rightarrow \infty} b(\gamma) = \lim_{\gamma \rightarrow \infty} f(\gamma) = 1$  and  $\lim_{\gamma \rightarrow \infty} \gamma f'(\gamma) = 0$  [8],  $\lim_{p_k \rightarrow \infty} \frac{\partial}{\partial p_k} \frac{f(p_k g)}{p_k + q_k} < 0$ , which, since (26) is continuous, implies that there is some  $p_k$  for which it is zero. This shows the existence of at least one  $\widehat{p}_k(g)$  in the concave region.

Finally, assume that there exist  $p_1$  and  $p_2$  such that  $p_2 \geq p_1 > p_0$  and  $\left. \frac{\partial}{\partial p_k} \frac{f(p_k g)}{p_k + q_k} \right|_{p_k=p_2} = \left. \frac{\partial}{\partial p_k} \frac{f(p_k g)}{p_k + q_k} \right|_{p_k=p_1} = 0$ . Then  $q_k g f'(gp_2) - b(gp_2) = q_k g f'(gp_1) - b(gp_1)$  or  $q_k g (f'(gp_2) - f'(gp_1)) = b(gp_2) - b(gp_1)$ . Since  $gp_2 > gp_1 > \gamma_0$ , the left side of this equation is non-positive while the right is non-negative [8], which is a contradiction unless  $p_1 = p_2$ , which means that the value of  $\widehat{p}_k(g)$  is unique. ■

**Lemma III.2.** For any user  $k$ , positive scalar  $q_k$ , fixed set of powers  $\mathbf{p}$ , and linear receiver discussed above, there is a unique vector  $\widetilde{\mathbf{p}}(\mathbf{p})$  that satisfies for each  $k$

$$\left. \frac{\partial}{\partial p} \frac{f(\gamma_k(p, \mathbf{p}_{-k}))}{p + q_k} \right|_{p=\widetilde{p}_k} = 0 \quad (28)$$

and it occurs in the concave region of the efficiency function:  $\widetilde{p}_k(\mathbf{p}) > p_0 \forall \mathbf{p} \in \mathbb{R}_+^K$ .

*Proof:* Let  $g = g_k(\mathbf{p}_{-k})$ . Then, by Lemma III.1, for each  $k$  there is a unique positive scalar  $\widehat{p}_k(g)$  that satisfies

$$\left. \frac{\partial}{\partial p} \frac{f(pg)}{p + q_k} \right|_{p=\widehat{p}_k(g)} = 0 \quad (29)$$

and it occurs in the concave region of the efficiency function:  $g\widehat{p}_k(g) > \gamma_0 \forall g \in \mathbb{R}_+$ . Let the elements of the vector  $\widetilde{\mathbf{p}}(\mathbf{p})$  be defined as  $\widetilde{p}_k(\mathbf{p}) = \widehat{p}_k(g_k(\mathbf{p}_{-k}))$  and the proof is complete. ■

**Lemma III.3.**  $\widehat{p}_k(g)$  is the unique number that satisfies

$$(\widehat{p}_k(g) + q_k)gf'(\widehat{p}_k(g)g) = f(\widehat{p}_k(g)g). \quad (30)$$

*Proof:* By Lemma III.2,  $\widehat{p}_k(g)$  is the unique number that satisfies Equation (21). So

$$0 = \frac{\partial}{\partial p} \frac{f(pg)}{p + q_k} \Big|_{p=\widehat{p}_k(g)} \quad (31)$$

$$= \frac{\partial}{\partial p} \frac{(p + q_k)gf'(pg) - f(pg)}{(p + q_k)^2} \Big|_{p=\widehat{p}_k(g)}, \quad (32)$$

which implies that  $\widehat{p}_k(g)$  satisfies

$$(\widehat{p}_k(g) + q_k)gf'(\widehat{p}_k(g)g) = f(\widehat{p}_k(g)g). \quad (33)$$

If  $\widehat{p}_k(g)$  satisfies Equation (30) then  $\widehat{p}_k(g)$  satisfies Equation (21), implying that there must be a unique solution to Equation (30). ■

The following theorem says that the game has at least one Nash equilibrium.

**Theorem III.4.** A Nash equilibrium exists for  $\mathcal{G}_r$ . Furthermore, if  $\mathbf{p}'$  is a Nash equilibrium point, for all users  $k$ :

$$p'_k = \begin{cases} \widetilde{p}_k(\mathbf{p}'), & \widetilde{p}_k(\mathbf{p}') \leq P_{\max}, \\ P_{\max}, & \text{else.} \end{cases} \quad (34)$$

*Proof:* Since  $[0, P_{\max}]$  is a nonempty, convex, and compact subset of some Euclidean space, a Nash equilibrium exists in  $\mathcal{G}_r$  if, for all  $k = 1, \dots, K$ ,  $u_k$  is continuous in  $\mathbf{p}$  and quasi-concave in  $p_k$  [2]. Choose any  $k \in \mathcal{K}$ . It is obvious that  $u_k = \frac{L}{M} R \frac{f(\gamma'_k)}{p_k + q_k}$  is continuous in  $\mathbf{p}$  since  $f(\cdot)$  is continuous and  $\gamma'_k$  is continuous in  $\mathbf{p}$ . Finally, Lemma III.2 shows that the root of  $u'_k(p_k)$  is unique, so, since  $u_k(p_k)$  is increasing before  $\widetilde{p}_k(\mathbf{p})$  and decreasing afterwards, this is a unique local maximum and thus a global maximum, implying that  $u_k$  is quasi-concave in  $p_k$ .

If  $\mathbf{p}'$  is a Nash equilibrium point, by definition  $\widetilde{p}_k(\mathbf{p}') = p'_k$  unless the Nash equilibrium involved a boundary point, that is that  $p'_k \in \{0, P_{\max}\}$ . For any vector  $\mathbf{p}_{-k}$ ,  $\frac{f(\gamma'_k(p_k, \mathbf{p}_{-k}))}{p_k + q_k} \Big|_{p_k=0} = 0$ , while there exists some finite  $p_k > 0$  such that  $f(\gamma'_k(p_k, \mathbf{p}_{-k})) > 0$  which implies that  $\frac{f(\gamma'_k(p_k, \mathbf{p}_{-k}))}{p_k + q_k} > 0$ , so the Nash equilibrium cannot occur with  $p'_k = 0$ . If  $\widetilde{p}_k(\mathbf{p}') > P_{\max}$  then  $u_k(p_k)$  is strictly increasing on  $[0, P_{\max}]$  so the optimal power for the  $k$ th user is  $P_{\max}$ . If  $\widetilde{p}_k(\mathbf{p}') < P_{\max}$  then  $u_k(p_k)$  is strictly decreasing for  $p_k > \widetilde{p}_k(\mathbf{p}')$ , so the optimal power for the  $k$ th user is  $\widetilde{p}_k(\mathbf{p}')$ . ■

From this point on, for simplicity we will assume that  $p'_k < P_{\max} \forall k$ ; the extension to the case where this does not hold is straightforward.

If the combination of interference and noise encountered by a user increases, this user's optimal SINR decreases. This is formally stated in the following lemma.

**Lemma III.5.** *If  $\mathbf{p}$  and  $\mathbf{p}'$  are non-negative  $K$ -vectors with the property that  $g_k(\mathbf{p}_{-k}) < g_k(\mathbf{p}'_{-k})$ , then  $\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k}) \leq \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})$ .*

*Proof:* Assume that  $\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k}) > \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})$ . Then

$$0 > (g_k(\mathbf{p}_{-k})q_k - g_k(\mathbf{p}'_{-k})q_k) f'(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) \quad (35)$$

$$= (g_k(\mathbf{p}_{-k})q_k + \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) f'(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) - (g_k(\mathbf{p}'_{-k})q_k + \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) f'(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) \quad (36)$$

$$= (g_k(\mathbf{p}_{-k})q_k + \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) f'(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) \quad (37)$$

$$\geq (g_k(\mathbf{p}_{-k})q_k + \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) f'(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) \quad (38)$$

$$= (g_k(\mathbf{p}_{-k})q_k + \bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) f'(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - (\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k}) - \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) f'(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) \quad (39)$$

$$= f(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) + (\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k}) - \bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) f'(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) \quad (40)$$

$$\geq f(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})) + (\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k}) - \bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) \frac{f(\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k})) - f(\bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k}))}{\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k}) - \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})} \quad (41)$$

$$= 0. \quad (42)$$

Since this is a contradiction,  $\bar{p}_k(\mathbf{p})g_k(\mathbf{p}_{-k}) \leq \bar{p}_k(\mathbf{p}')g_k(\mathbf{p}'_{-k})$ . ■

**Lemma III.6.**  $\frac{\partial}{\partial g} \widehat{p}_k(g) \leq 0 \forall g \in \mathbb{R}_+, \forall k \Leftrightarrow \mathbf{p} \geq \mathbf{p}' \Rightarrow \bar{\mathbf{p}}(\mathbf{p}) \geq \bar{\mathbf{p}}(\mathbf{p}')$

*Proof:* If  $\frac{\partial}{\partial g} \widehat{p}_k(g) < 0 \forall g \in \mathbb{R}_+, \forall k$  then each of the  $\widehat{p}_k(g)$  functions are decreasing, so, since  $\mathbf{p} \geq \mathbf{p}' \Rightarrow g_k(\mathbf{p}_{-k}) \leq g_k(\mathbf{p}'_{-k}) \forall k$ ,  $\bar{p}_k(\mathbf{p}) = \widehat{p}_k(g_k(\mathbf{p}_{-k})) \geq \widehat{p}_k(g_k(\mathbf{p}'_{-k})) = \bar{p}_k(\mathbf{p}') \forall k$ .

If  $\mathbf{p} \geq \mathbf{p}' \Rightarrow \bar{\mathbf{p}}(\mathbf{p}) \geq \bar{\mathbf{p}}(\mathbf{p}')$  then it must be that  $g \leq g' \Rightarrow \widehat{p}_k(g) \geq \widehat{p}_k(g') \forall k$ , which means that  $\frac{\partial}{\partial g} \widehat{p}_k(g) \leq 0 \forall g \in \mathbb{R}_+, \forall k$ . ■

Note that in the case that  $q_k = 0$ , Equation (30) simplifies to  $\gamma f'(\gamma) = f(\gamma)$ , as expected.

**Lemma III.7.**  $q_k f'(\widehat{p}_k(g)g) + g \widehat{p}_k(g) (\widehat{p}_k(g) + q_k) f''(\widehat{p}_k(g)g) < 0 \forall g \in \mathbb{R}_+ \Leftrightarrow \frac{\partial}{\partial g} \widehat{p}_k(g) < 0 \forall g \in \mathbb{R}_+$

*Proof:* Taking the derivative of both sides of Equation (30) with respect to  $g$  results in the equation

$$\begin{aligned} & \left( \frac{\partial}{\partial g} \widehat{p}_k(g) \right) g f'(\widehat{p}_k(g)g) + (\widehat{p}_k(g) + q_k) f'(\widehat{p}_k(g)g) + (\widehat{p}_k(g) + q_k) g \left( \left( \frac{\partial}{\partial g} \widehat{p}_k(g) \right) g + \widehat{p}_k(g) \right) f''(\widehat{p}_k(g)g) \\ &= \left( \left( \frac{\partial}{\partial g} \widehat{p}_k(g) \right) g + \widehat{p}_k(g) \right) f'(\widehat{p}_k(g)g), \end{aligned} \quad (43)$$

which simplifies to

$$q_k f'(\widehat{p}_k(g)g) + (\widehat{p}_k(g) + q_k) g \left( \left( \frac{\partial}{\partial g} \widehat{p}_k(g) \right) g + \widehat{p}_k(g) \right) f''(\widehat{p}_k(g)g) = 0, \quad (44)$$

implying that

$$\frac{\partial}{\partial g} \widehat{p}_k(g) = - \frac{q_k f'(\widehat{p}_k(g)g) + (\widehat{p}_k(g) + q_k) g \widehat{p}_k(g) f''(\widehat{p}_k(g)g)}{(\widehat{p}_k(g) + q_k) g f''(\widehat{p}_k(g)g)}. \quad (45)$$

Since  $g\widehat{p}_k(g) > \gamma_0 \forall g \in \mathbb{R}_+$ , as was shown in Lemma III.1,  $f''(\widehat{p}_k(g)g) < 0$ . Therefore,  $\frac{\partial}{\partial g} \widehat{p}_k(g)$  is negative if and only if the numerator in Equation (45) is negative, which is the condition given in the lemma statement. ■

**Theorem III.8.** *The Nash equilibrium for  $\mathcal{G}_r$  is unique if any of the following conditions hold:*

- 1)  $\mathbf{p} \geq \mathbf{p}' \implies \widetilde{\mathbf{p}}(\mathbf{p}) \geq \widetilde{\mathbf{p}}(\mathbf{p}')$ ,
- 2)  $\frac{\partial}{\partial g} \widehat{p}_k(g) \leq 0 \forall g \in \mathbb{R}_+ \forall k$ , or
- 3)  $q_k f'(\widehat{p}_k(g)g) + g\widehat{p}_k(g)(\widehat{p}_k(g) + q_k) f''(\widehat{p}_k(g)g) \leq 0 \forall g \in \mathbb{R}_+ \forall k$ .

Furthermore, the algorithm where, for each time  $t$ , the users use the power described by  $\mathbf{p}(t) = \widetilde{\mathbf{p}}(\mathbf{p}(t-1))$  converges to the unique Nash equilibrium for any initial choice of power vector.

*Proof:* The Nash equilibrium is unique and the described algorithm converges to it if  $\widetilde{\mathbf{p}}(\mathbf{p})$  satisfies the following three properties [9]:

- 1) positivity:  $\widetilde{\mathbf{p}}(\mathbf{p}) > 0$  for all  $\mathbf{p} \geq 0$ ;
- 2) monotonicity:  $\mathbf{p} \geq \mathbf{p}' \implies \widetilde{\mathbf{p}}(\mathbf{p}) \geq \widetilde{\mathbf{p}}(\mathbf{p}')$ ;
- 3) scalability:  $\mu > 1 \implies \mu \widetilde{\mathbf{p}}(\mathbf{p}) > \widetilde{\mathbf{p}}(\mu \mathbf{p})$ .

Positivity is obvious since we already showed that  $\widetilde{p}_k(\mathbf{p}) > p_0 > 0$  for all  $k$ . By Lemmas III.6 and III.7, monotonicity is assured when any of the (equivalent) conditions in the theorem statement are true.

For scalability, for any  $\mu > 1$ , since for the receivers considered here,  $\mu g(\mu \mathbf{p}_{-k}) > g(\mathbf{p}_{-k})$ , Lemma III.5 implies that  $\widetilde{\mathbf{p}}(\mathbf{p}) > \frac{g(\mu \mathbf{p}_{-k})}{g(\mathbf{p}_{-k})} \widetilde{\mathbf{p}}(\mu \mathbf{p}) > \frac{1}{\mu} \widetilde{\mathbf{p}}(\mu \mathbf{p})$ . ■

The solution to  $\gamma f'(\gamma) = f(\gamma)$  is a lower bound on  $\widehat{p}_k(g)$ . In fact, as  $q_k$  increases, so does  $\widehat{p}_k(g)$ . That is, as the power necessary to run increases, the nodes aim for a higher SINR: to make the transmission worthwhile, they need more throughput.

**Lemma III.9.** For set values of  $g_k$ ,

$$\frac{\partial \widehat{p}_k}{\partial q_k} > 0. \quad (46)$$

*Proof:* From Lemma III.3,  $\widehat{p}_k$  is the unique number that satisfies

$$(\widehat{p}_k + q_k)g_k f'(\widehat{p}_k g_k) = f(\widehat{p}_k g_k). \quad (47)$$

Taking the derivative of both sides of this equation with respect to  $q_k$  gives

$$\left( \frac{\partial \widehat{p}_k}{\partial q_k} + 1 \right) g_k f'(\widehat{p}_k g_k) + (\widehat{p}_k + q_k) \frac{\partial \widehat{p}_k}{\partial q_k} g_k^2 f''(\widehat{p}_k g_k) = \frac{\partial \widehat{p}_k}{\partial q_k} g_k f'(\widehat{p}_k g_k), \quad (48)$$

which means that

$$\frac{\partial \widehat{p}_k}{\partial q_k} = - \frac{f'(\widehat{p}_k g_k)}{(\widehat{p}_k + q_k)g_k f''(\widehat{p}_k g_k)} > 0. \quad (49)$$

■

**Lemma III.10.**  $\frac{\partial u_k}{\partial g_k} > 0$ .

*Proof:* Consider what happens if  $g_k$  is changed from  $g_k$  to  $g_k + \alpha$  for some  $\alpha > 0$ . Then the utility of user  $k$  is

$$u_k(g_k + \alpha) = \frac{f(\widehat{p}_k(g_k + \alpha)(g_k + \alpha))}{\widehat{p}_k(g_k + \alpha) + q_k} \quad (50)$$

$$\geq \frac{f(\widehat{p}_k(g_k)(g_k + \alpha))}{\widehat{p}_k(g_k) + q_k} \quad (51)$$

$$> \frac{f(\widehat{p}_k(g_k)g_k)}{\widehat{p}_k(g_k) + q_k} \quad (52)$$

$$= u_k(g_k), \quad (53)$$

where the first inequality comes from the fact that  $\widehat{p}_k$  is chosen optimally and the second because  $f(\cdot)$  is an increasing function. ■

**Theorem III.11.** For any user  $j$  (including  $j = k$ ),  $\frac{\partial u_j}{\partial g_k} \leq 0$ , where the inequality is strict for the MF and MMSE receivers and for the decorrelator when  $j = k$ .

*Proof:* Consider the algorithm described in Theorem III.8 that converges to the Nash equilibrium. If the starting point ( $\mathbf{p}(0)$ ) were taken as the power vector at Nash equilibrium for some value of  $q_k$  and  $q_k$  were increased slightly, then for  $l \neq k$ ,  $p_l(1) = p_l(0)$  while  $p_k(1) > p_k(0)$  by Lemma III.9. For the decorrelating receiver, this is the new Nash equilibrium. For the MF or MMSE receiver, for all users  $l \neq k$ , their values for  $g_l$  at time 2 are strictly less than their  $g_l$  values at time 1; by monotonicity this

means that  $p_l(2) \geq p_l(1)$ . This process continues with each user increasing its power at each time until the algorithm converges (which it is guaranteed to do by Theorem III.8). Therefore, with a MF or MMSE receiver, any user  $j$  (and user  $k$  in the case of a DE receiver) has a lower value of  $g_j$ , and thus, by Lemma III.10 a lower utility, when  $q_k$  is increased. ■

Except in the case of the decorrelator, the non-cooperative result is not socially optimal, as the following theorem shows.

**Theorem III.12.** *For the decorrelator, the Nash equilibrium is Pareto optimal. For the MMSE and MF receivers, the Nash equilibrium is not Pareto optimal and can be improved upon if every user decreases its power by a small factor.*

*Proof:* The decorrelator cancels out all interference; the receiver and its performance are both independent of the powers of the interfering nodes. This means that the different nodes games' are independent and thus the Nash equilibrium is Pareto optimal.

For the matched filter and MMSE receivers, consider what happens when the power vector is  $\mathbf{p} = \alpha \hat{\mathbf{p}}$ , where  $\hat{\mathbf{p}}$  is the power vector of the Nash equilibrium and  $\alpha$  is a positive number. Specifically, consider the derivative of the  $k$ th user's utility with respect to  $\alpha$  at the point where  $\alpha = 1$  (that is, at the Nash equilibrium). For both of these receivers, this derivative is negative for all  $k$ : there exists a factor by which each user can increase its power from the Nash equilibrium power so that all users will increase their utilities.

$$\left. \frac{\partial u_k^{\text{MF}}}{\partial \alpha} \right|_{\alpha=1} = \frac{LR(p_k + q_k) \frac{\partial \gamma_k^{\text{MF}}}{\partial \alpha} f'(\gamma_k^{\text{MF}}) - \frac{\partial p_k}{\partial \alpha} f(\gamma_k^{\text{MF}})}{M(p_k + q_k)^2} \Big|_{\alpha=1} \quad (54)$$

$$= \frac{LR(\bar{p}_k + q_k) \frac{\sigma^2 \bar{p}_k h_j^{m(k)2}}{(\sigma^2 + \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \rho_{kj}^2)} f'(\gamma_k^{\text{MF}}(\bar{\mathbf{p}})) - \bar{p}_k f(\gamma_k^{\text{MF}}(\bar{\mathbf{p}}))}{M(\bar{p}_k + q_k)^2} \quad (55)$$

$$= \frac{LR \frac{\sigma^2 \bar{p}_k}{\sigma^2 + \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \rho_{kj}^2} f(\gamma_k^{\text{MF}}(\bar{\mathbf{p}})) - \bar{p}_k f(\gamma_k^{\text{MF}}(\bar{\mathbf{p}}))}{M(\bar{p}_k + q_k)^2} \quad (56)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MF}}(\bar{\mathbf{p}}))}{M(\bar{p}_k + q_k)^2} \left( \frac{\sigma^2}{\sigma^2 + \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \rho_{kj}^2} - 1 \right) \quad (57)$$

$$= -\frac{LR \bar{p}_k f(\gamma_k^{\text{MF}}(\bar{\mathbf{p}})) \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \rho_{kj}^2}{M(\bar{p}_k + q_k)^2 (\sigma^2 + \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \rho_{kj}^2)} \quad (58)$$

$$< 0, \quad (59)$$

$$\left. \frac{\partial u_k^{\text{MMSE}}}{\partial \alpha} \right|_{\alpha=1} = \frac{LR(p_k + q_k) \frac{\partial \gamma_k^{\text{MMSE}}}{\partial \alpha} f'(\gamma_k^{\text{MMSE}}) - \frac{\partial p_k}{\partial \alpha} f(\gamma_k^{\text{MMSE}})}{M(p_k + q_k)^2} \Big|_{\alpha=1} \quad (60)$$

$$= \frac{LR(\bar{p}_k + q_k) \sigma^2 \bar{p}_k h_k^{m(k)2} \mathbf{s}_k^T \mathbf{A}_k^{-2} \mathbf{s}_k f'(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) - \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}}))}{M(\bar{p}_k + q_k)^2} \quad (61)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) (\sigma^2 \mathbf{s}_k^T \mathbf{A}_k^{-2} \mathbf{s}_k - \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k)}{M(\bar{p}_k + q_k)^2 \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \quad (62)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) \mathbf{s}_k^T \mathbf{A}_k^{-1} (\sigma^2 \mathbf{I} - \mathbf{A}_k) \mathbf{A}_k^{-1} \mathbf{s}_k}{M(\bar{p}_k + q_k)^2 \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \quad (63)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) \mathbf{s}_k^T \mathbf{A}_k^{-1} \left( \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{A}_k^{-1} \mathbf{s}_k}{M(\bar{p}_k + q_k)^2 \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \quad (64)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) \left( \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_j \mathbf{s}_j^T \mathbf{A}_k^{-1} \mathbf{s}_k \right)}{M(\bar{p}_k + q_k)^2 \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \quad (65)$$

$$= \frac{LR \bar{p}_k f(\gamma_k^{\text{MMSE}}(\bar{\mathbf{p}})) \left( \sum_{j \neq k} \bar{p}_j h_j^{m(k)2} (\mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_j)^2 \right)}{M(\bar{p}_k + q_k)^2 \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k} \quad (66)$$

$$< 0, \quad (67)$$

where Equations (56) and (62) come from Lemma III.3 and Equations (55) and (61) make use of the



following simplifications:

$$\left. \frac{\partial \gamma_k^{\text{MF}}}{\partial \alpha} \right|_{\alpha=1} = \left. \frac{\partial}{\partial \alpha} \frac{p_k h_k^{m(k)^2}}{\sigma^2 + \sum_{j \neq k} p_j h_j^{m(k)^2} \rho_{kj}^2} \right|_{\alpha=1} \quad (68)$$

$$= \left. \frac{\left( \sigma^2 + \sum_{j \neq k} p_j h_j^{m(k)^2} \rho_{kj}^2 \right) \frac{\partial p_k}{\partial \alpha} h_k^{m(k)^2} - p_k h_k^{m(k)^2} \sum_{j \neq k} \frac{\partial p_j}{\partial \alpha} h_j^{m(k)^2} \rho_{kj}^2}{\left( \sigma^2 + \sum_{j \neq k} p_j h_j^{m(k)^2} \rho_{kj}^2 \right)^2} \right|_{\alpha=1} \quad (69)$$

$$= \left. \frac{\left( \sigma^2 + \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \rho_{kj}^2 \right) \widehat{p}_k h_k^{m(k)^2} - \widehat{p}_k h_k^{m(k)^2} \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \rho_{kj}^2}{\left( \sigma^2 + \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \rho_{kj}^2 \right)^2} \right|_{\alpha=1} \quad (70)$$

$$= \frac{\sigma^2 \widehat{p}_k h_k^{m(k)^2}}{\left( \sigma^2 + \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \rho_{kj}^2 \right)^2}, \quad (71)$$

and

$$\left. \frac{\partial \gamma_k^{\text{MMSE}}}{\partial \alpha} \right|_{\alpha=1} = \left. \frac{\partial p_k}{\partial \alpha} h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k - p_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \frac{\partial \mathbf{A}_k}{\partial \alpha} \mathbf{A}_k^{-1} \mathbf{s}_k \right|_{\alpha=1} \quad (72)$$

$$= \left. \frac{\partial p_k}{\partial \alpha} h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k - p_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \left( \sum_{j \neq k} \frac{\partial p_j}{\partial \alpha} h_j^{m(k)^2} \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{A}_k^{-1} \mathbf{s}_k \right|_{\alpha=1} \quad (73)$$

$$= \widehat{p}_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \mathbf{s}_k - \widehat{p}_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \left( \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{A}_k^{-1} \mathbf{s}_k \quad (74)$$

$$= \widehat{p}_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-1} \left( \mathbf{A}_k - \sum_{j \neq k} \widehat{p}_j h_j^{m(k)^2} \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{A}_k^{-1} \mathbf{s}_k \quad (75)$$

$$= \sigma^2 \widehat{p}_k h_k^{m(k)^2} \mathbf{s}_k^T \mathbf{A}_k^{-2} \mathbf{s}_k. \quad (76)$$

■

#### A. Nash equilibrium for a specific efficiency function

Consider the efficiency function  $f(\gamma) = (1 - e^{-\gamma})^M$ . For this function

$$\begin{aligned} & q_k f'(\widehat{p}_k(g)g) + g \widehat{p}_k(g) (\widehat{p}_k(g) + q_k) f''(\widehat{p}_k(g)g) \\ &= M e^{-\widehat{p}_k(g)g} (1 - e^{-\widehat{p}_k(g)g})^{M-2} (q_k (1 - e^{-\widehat{p}_k(g)g}) + \widehat{p}_k(g)g (\widehat{p}_k(g) + q_k) (M e^{-\widehat{p}_k(g)g} - 1)) \end{aligned} \quad (77)$$

$$\begin{aligned} &= M e^{-\widehat{p}_k(g)g} (1 - e^{-\widehat{p}_k(g)g})^{M-2} \\ &\cdot \left( q_k (1 - 2e^{-\widehat{p}_k(g)g} + (M \widehat{p}_k(g)g + 1) e^{-\widehat{p}_k(g)g} - \widehat{p}_k(g)g) + \widehat{p}_k(g)^2 g (M e^{-\widehat{p}_k(g)g} - 1) \right) \end{aligned} \quad (78)$$

$$< M e^{-\widehat{p}_k(g)g} (1 - e^{-\widehat{p}_k(g)g})^{M-2} \left( q_k (2 - 2e^{-\widehat{p}_k(g)g} - \widehat{p}_k(g)g) + \widehat{p}_k(g)^2 g (M e^{-\widehat{p}_k(g)g} - 1) \right), \quad (79)$$

where the last inequality comes because, by Lemma III.9,  $\widehat{p}_k(g)g > \gamma^*$ , where  $\gamma^*$  is the solution to  $\gamma f'(\gamma) = f(\gamma)$ . Then, since  $\widehat{p}_k(g)g > \gamma^* > \gamma_0$ ,  $b(\widehat{p}_k(g)g) > b(\gamma^*) = 0$ , so  $f(\widehat{p}_k(g)g) > \widehat{p}_k(g)g f'(\widehat{p}_k(g)g)$  which, for  $f(\gamma) = (1 - e^{-\gamma})^M$ , implies that  $(M\widehat{p}_k(g)g + 1)e^{-\widehat{p}_k(g)g} < 1$ .

Since  $\widehat{p}_k(g)g > \gamma_0$  and  $f''(\gamma_0) = 0 \implies Me^{-\gamma_0} = 1$ ,  $Me^{-\widehat{p}_k(g)g} < 1$  and  $\widehat{p}_k(g)g > \log(M)$ . For  $M \geq 5$ ,  $\widehat{p}_k(g)g > \log(M) > 1.6$ , which means that  $2 - 2e^{-\widehat{p}_k(g)g} - \widehat{p}_k(g)g < 0$ . This means that, for  $M \geq 5$ , Equation (79) is negative, so Theorem III.8 holds and there is a unique Nash equilibrium.

#### IV. NUMERICAL RESULTS

Consider a multi-hop network with  $K$  nodes distributed randomly in a square whose area is  $100K$  square km, surrounding an access point in the center. For simplicity, the simulations assume a routing scheme where all nodes transmit to the closest node that is closer to the access point (or the access point of that is closest). The packets each contain 100 bits of data and no overhead ( $L = M = 100$ ); the transmission rate is  $R = 100$  kb/s; the thermal noise power is  $\sigma^2 = 5 \times 10^{-16}$  Watts; the channel gains are distributed with a Rayleigh distribution with mean  $0.3d^{-2}$ , where  $d$  is the distance between the transmitter and receiver; and the processing gain is  $N = 32$ . We use the same efficiency function as [5], namely  $f(\gamma) = (1 - e^{-\gamma})^M$ , which can be shown to satisfy the conditions for the existence of a unique Nash equilibrium. Finally, the amount of energy that a node has to expend to run,  $q_k$ , is assumed to be the same for all nodes and is allow to range from 0.0001 Joules to 1 Joule per transmission (equivalently, for the rate and packet size given, it ranges between 0.001 Watts and 10 Watts) while  $P_{\max}$  is taken to be infinite.

Figure 1 shows the mean utility (averaged over 100 realizations of the system) for the different receivers as a function of the system load. Here the load ranges from 0 to 1.5. For loads greater than 1, the DE receiver cannot be used. Changing the value of  $q$  by a factor of ten for all the users changes the mean utility by roughly a factor of ten as well. As is shown in Figure 1, for very low loads, all the receivers perform similarly well, with the various utilities at low load depending only on the value of  $q$ . As the load increases, the MF receiver is most affected, with the mean utility dropping by about a factor of ten when the load increases from 10% to 50%. The performance of the DE and MMSE receivers are similar, although the MMSE receiver outperforms the DE receiver at all points, with more significant gains at high load.

As shown above, for the MMSE and MF receivers, the Nash equilibrium point is not Pareto optimal. This is particularly obvious for the matched filter. Figure 2 shows how the utility for all 16 users in one scenario change with time when the simple algorithm described above is run. Notice that all users have

better utility before convergence. In fact, for this example,  $\max_k\{u_k\}$  achieves its maximum at  $t = 2$  while  $\min_k\{u_k\}$  achieves its maximum at  $t = 6$ . Figure 3 shows the transmit power for all users in the same example; from this it is clear that the users are transmitting at higher and higher powers to the detriment of their utilities. The MMSE receiver tends to do much better (though it would still be better for all users to use slightly less power each); the MMSE receiver also converges much faster. The DE receiver converges in just one time step (due to the independence of the power choices of different users) and results in a Pareto optimal solution.

## V. CONCLUSION

In this paper, we have analyzed the cross-layer design issue of energy efficient communication in multi-hop networks using a game theoretic model. We have extended previous work in this area to consider the energy costs used in operating the receiver and transmitter, in addition to the transmit costs. A noncooperative game has been proposed in which users are allowed to choose their transmit powers and uplink receivers to maximize their utilities. We have seen that, amongst all linear receivers, the MMSE receiver is optimal and there is a significant improvement in achieved utility when multiuser detectors are used in place of the conventional MF, particularly in systems with high loads. We have shown that, for the MF, MMSE, and DE receivers, a unique Nash equilibrium exists. For the decorrelator, the Nash equilibrium is Pareto optimal, but for the MF and MMSE receivers, this Nash equilibrium is not Pareto optimal and can be improved upon if each user transmits at slightly less power. Future work will extend this analysis to treat more complex systems and to allow the nodes more strategy options.

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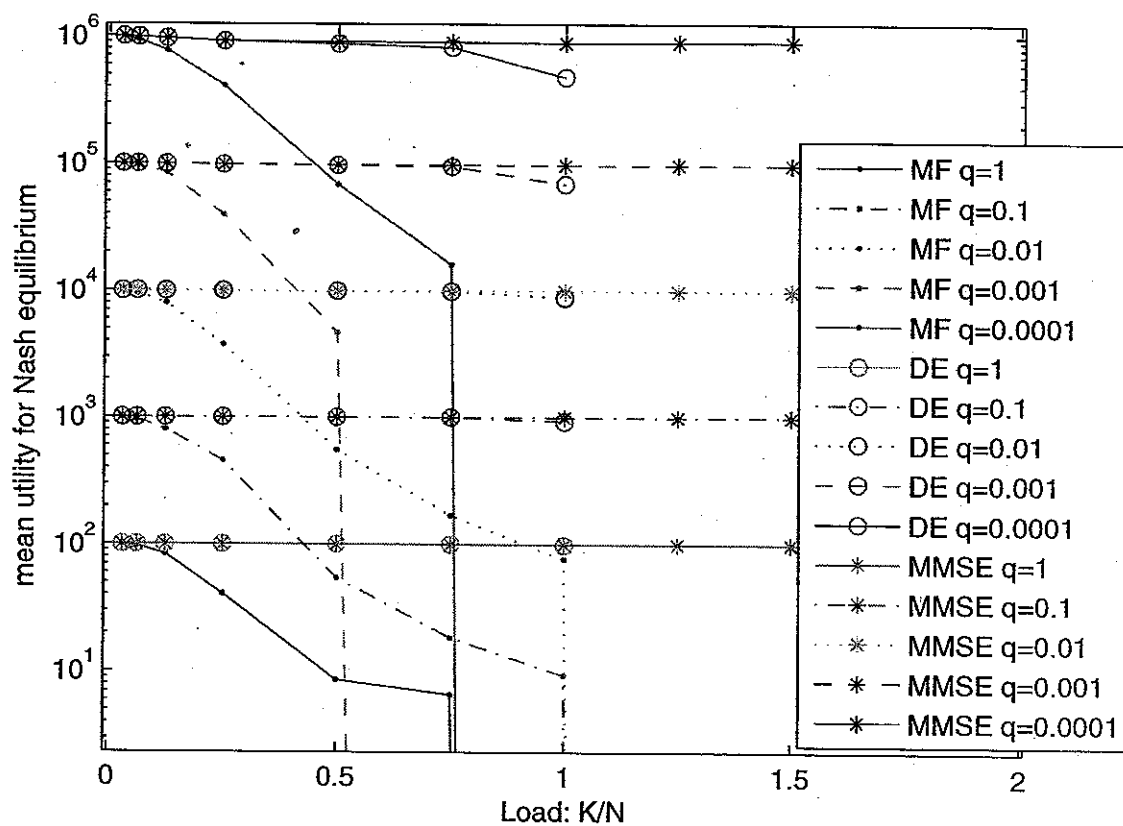


Fig. 1. Mean utility for the different receivers

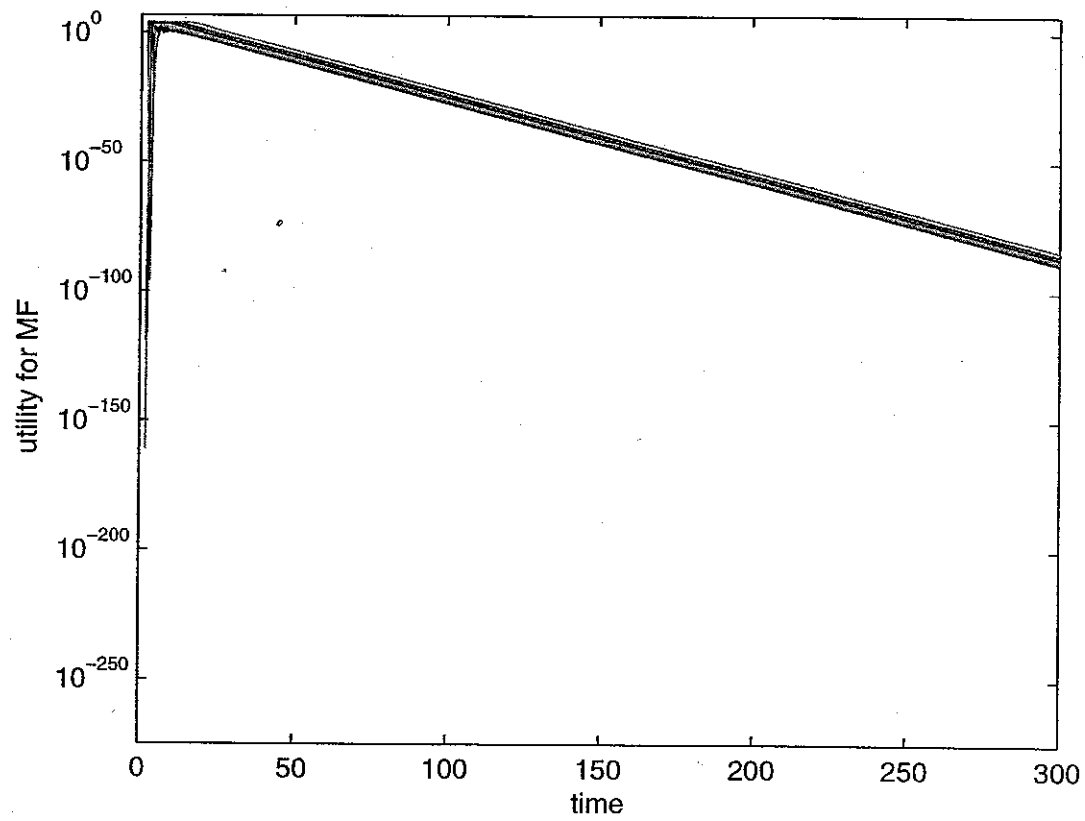


Fig. 2. Utility for all 16 users using the MF for one scenario with  $\beta = \frac{1}{2}$  and  $q = 0.01$

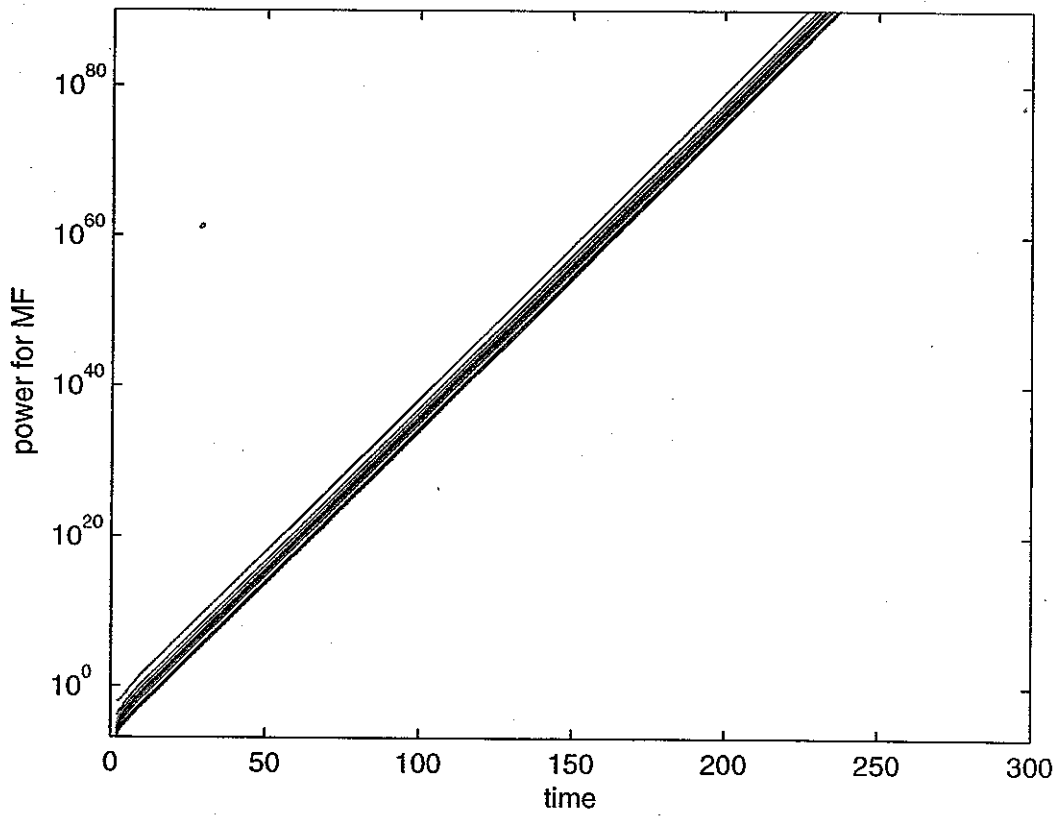


Fig. 3. Power for all 16 users using the MF for one scenario with  $\beta = \frac{1}{2}$  and  $q = 0.01$

# LARGE SYSTEM ANALYSIS OF GAME-THEORETIC POWER CONTROL IN UWB WIRELESS NETWORKS WITH RAKE RECEIVERS

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## ABSTRACT

This paper studies the performance of partial-Rake (PRake) receivers in impulse-radio ultrawideband wireless networks when an energy-efficient power control scheme is adopted. Due to the large bandwidth of the system, the multipath channel is assumed to be frequency-selective. By using noncooperative game-theoretic models and large system analysis, explicit expressions are derived in terms of network parameters to measure the effects of self- and multiple-access interference at a receiving access point. Performance of the PRake is compared in terms of achieved utilities and loss to that of the all-Rake receiver.

## 1. INTRODUCTION

Ultrawideband (UWB) technology is considered to be a potential candidate for next-generation multiuser data networks, due to its large spreading factor (which implies large multiuser capacity) and its lower spectral density (which allows coexistence with incumbent systems). The requirements for designing high-speed data mobile terminals include efficient resource allocation and interference reduction. These issues aim to allow each user to achieve the required quality of service at the uplink receiver without causing unnecessary interference to other users in the system, and minimizing power consumption. Scalable energy-efficient power control (PC) techniques can be derived using game theory [1, 2].

In this work, performance of partial Rake (PRake) receivers [3] is studied in terms of transmit powers and utilities achieved in the uplink of an infrastructure network at the Nash equilibrium, where utility here is defined as the ratio of throughput to transmit power. By using the large system analysis proposed in [2], we obtain a general characterization for the terms due to self-interference (SI) and multiple access interference (MAI). Explicit expressions for the utilities achieved at the Nash equilibrium are then derived, and an approximation for the loss of PRake receivers with respect to (wrt) all-Rake (ARake) receivers is proposed.

\*This research was supported in part by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252, and in part by the U. S. Defense Advanced Research Projects Agency under Grant HR0011-06-1-0052.

The remainder of the paper is organized as follows. Some background for this work is given in Sect. 2, where the system model and the results of the game-theoretic PC approach are shown. In Sect. 3, large system analysis is used to evaluate the effects of the interference at the Nash equilibrium. Performance of the PRake at the Nash equilibrium is analyzed in Sect. 4, where also a comparison with simulations is provided. Some conclusions are drawn in Sect. 5.

## 2. BACKGROUND

### 2.1. System Model

Commonly, impulse-radio (IR) systems are employed to implement UWB systems. We focus here on a binary phase shift keying (BPSK) time hopping (TH) IR-UWB system with polarity randomization [4]. A network with  $K$  users transmitting to a common concentration point is considered. The processing gain is  $N = N_f \cdot N_c$ , where  $N_f$  is the number of pulses that represent one information symbol, and  $N_c$  is the number of possible pulse positions in a frame [4]. The transmission is assumed to be over *frequency selective channels*, with the channel for user  $k$  modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^L \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \quad (1)$$

where  $T_c$  is the duration of the transmitted UWB pulse;  $L$  is the number of channel paths;  $\alpha_k = [\alpha_1^{(k)}, \dots, \alpha_L^{(k)}]^T$  and  $\tau_k$  are the fading coefficients and the delay of user  $k$ , respectively. Considering a chip-synchronous scenario, the symbols are misaligned by an integer multiple of  $T_c$ :  $\tau_k = \Delta_k T_c$ , for every  $k$ , where  $\Delta_k$  is uniformly distributed in  $\{0, 1, \dots, N-1\}$ . In addition, we assume that the channel characteristics remain unchanged over a number of symbol intervals [4].

Due to high resolution of UWB signals, multipath channels can have hundreds of components, especially in indoor environments. To mitigate the effect of multipath fading as much as possible, we consider an access point where  $K$  Rake receivers [3] are used.<sup>1</sup> The Rake receiver for user  $k$  is in

<sup>1</sup>For ease of calculation, perfect channel estimation is considered throughout the paper.



general composed of  $L$  coefficients, where the vector  $\beta_k = \mathbf{G} \cdot \alpha_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$  represents the combining weights for user  $k$ , and the  $L \times L$  matrix  $\mathbf{G}$  depends on the type of Rake receiver employed. In particular, if  $\{\mathbf{G}\}_{ll} = 1$  for  $1 \leq l \leq r \cdot L$ , and 0 elsewhere, where  $r \triangleq L_P/L$  and  $0 < L_P \leq L$ , a PRake with  $L_P$  fingers using maximal ratio combining (MRC) scheme is considered. It is worth noting that, when  $r = 1$ , an ARake is implemented.

The signal-to-interference-plus-noise ratio (SINR) of the  $k$ th user at the output of the Rake receiver can be well approximated (for large  $N_f$ , typically at least 5) by [4]

$$\gamma_k = \frac{h_k^{(\text{SP})} p_k}{h_k^{(\text{SI})} p_k + \sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2}, \quad (2)$$

where  $\sigma^2$  is the variance of the additive white Gaussian noise (AWGN) at the receiver;  $p_k$  denotes the transmit power of user  $k$ ; and the gains are expressed by

$$h_k^{(\text{SP})} = \beta_k^H \cdot \alpha_k, \quad (3)$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2}{\beta_k^H \cdot \alpha_k}, \quad (4)$$

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\|\mathbf{B}_k^H \cdot \alpha_j\|^2 + \|\mathbf{A}_j^H \cdot \beta_k\|^2 + |\beta_k^H \cdot \alpha_j|^2}{\beta_k^H \cdot \alpha_k}, \quad (5)$$

where the matrices

$$\mathbf{A}_k = \begin{pmatrix} \alpha_L^{(k)} & \dots & \dots & \alpha_2^{(k)} \\ 0 & \alpha_L^{(k)} & \dots & \alpha_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \alpha_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (6)$$

$$\mathbf{B}_k = \begin{pmatrix} \beta_L^{(k)} & \dots & \dots & \beta_2^{(k)} \\ 0 & \beta_L^{(k)} & \dots & \beta_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \beta_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (7)$$

$$\Phi = \text{diag}\{\phi_1, \dots, \phi_{L-1}\}, \quad \phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}}, \quad (8)$$

have been introduced for convenience of notation.

## 2.2. The Game-Theoretic Power Control Game

Consider the application of noncooperative PC techniques to the wireless network described above. Focusing on mobile terminals, where it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput, a game-theoretic energy-efficient approach as the one described in [2] is considered.

We examine a noncooperative PC game in which each user seeks to maximize its own utility function as follows. Let  $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$  be the proposed game where  $\mathcal{K} = \{1, \dots, K\}$  is the index set for the users;  $P_k = [0, \bar{p}]$  is the strategy set, with  $\bar{p}$  denoting the maximum power constraint; and  $u_k(\mathbf{p})$  is the payoff function for user  $k$  [1]:

$$u_k(\mathbf{p}) = \frac{D}{M} R \frac{f(\gamma_k)}{p_k}, \quad (9)$$

where  $\mathbf{p} = [p_1, \dots, p_K]$  are the transmit powers;  $D$  is the number of information bits per packet;  $M$  is the total number of bits per packet;  $R$  is the transmission rate;  $\gamma_k$  is the SINR (2) for user  $k$ ; and  $f(\gamma_k)$  is the efficiency function representing the packet success rate (PSR), i.e., the probability that a packet is received without an error.

When the efficiency function is increasing, S-shaped [1], and continuously differentiable, with  $f(0) = 0$ ,  $f(+\infty) = 1$ ,  $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$ , it has been shown [2] that the solution of the maximization problem  $\max_{p_k \in P_k} u_k(\mathbf{p})$  is

$$p_k^* = \min \left\{ \frac{\gamma_k^* \left( \sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^* / \gamma_{0,k})}, \bar{p} \right\}, \quad (10)$$

where  $\gamma_k^*$  is the solution of

$$f'(\gamma_k^*) \gamma_k^* (1 - \gamma_k^* / \gamma_{0,k}) = f(\gamma_k^*), \quad (11)$$

with  $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma_k^*}$ , and  $\gamma_{0,k} = h_k^{(\text{SP})}/h_k^{(\text{SI})}$ .

In the typical case of multiuser UWB systems,  $N \gg K$ . If  $\bar{p}$  is sufficiently large, (10) can be reduced to [2]

$$p_k^* = \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \bar{\gamma}^*}{1 - \bar{\gamma}^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1})}, \quad (12)$$

where  $\zeta_k^{-1} = \sum_{j \neq k} h_{kj}^{(\text{MAI})}/h_j^{(\text{SP})}$ ; and  $\bar{\gamma}^*$  is the SINR at the Nash equilibrium in the absence of SI, i.e., it is the solution of (11) when  $\gamma_{0,k} = +\infty$ .

A necessary and sufficient condition for the Nash equilibrium to be achieved simultaneously by all the  $K$  users, and thus for (12) to be valid, is [2]

$$\bar{\gamma}^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1}) < 1 \quad \forall k \in \mathcal{K}. \quad (13)$$

## 3. ANALYSIS OF THE INTERFERENCE

### 3.1. Analytical Results

As can be verified in (12), the amount of transmit power  $p_k^*$  required to achieve the target SINR  $\gamma_k^*$  will depend not only on the gain  $h_k^{(\text{SP})}$ , but also on the SI term  $h_k^{(\text{SI})}$  (through  $\gamma_{0,k}$ ) and the interferers  $h_{kj}^{(\text{MAI})}$  (through  $\zeta_k$ ). To derive quantitative results for the transmit powers independent of SI and MAI terms, it is possible to resort to a large-system analysis [2].

For ease of calculation, the expressions derived in the remainder of the paper consider the following assumptions:

$$\nu(\Lambda, r, \rho) = \begin{cases} \frac{\Lambda(\Lambda^\rho - 1)(4\Lambda^{2r} + 3\Lambda^\rho - 1) - 2\Lambda^{r+\rho}(\Lambda^r + 3\Lambda - 1)\rho \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } 0 \leq \rho \leq \min(r, 1 - r); \\ \frac{\Lambda(4\Lambda^\rho - 1)(\Lambda^{2r} - 1) - 2\Lambda^{r+\rho}(3\Lambda^r - \rho + \Lambda^r \rho) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } r \leq \rho \leq 1 - r \text{ and } r \leq 1/2; \\ \frac{-4\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} + 3\Lambda^{2+2\rho} - 2\Lambda^{1+r+\rho}(r + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } 1 - r \leq \rho \leq r \text{ and } r \geq 1/2; \\ \frac{-\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} - 2\Lambda^{1+r+\rho}(r + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } \max(r, 1 - r) \leq \rho \leq 1; \\ \frac{2\Lambda(\Lambda^{2r} - 1) - (\Lambda^r + r + 3\Lambda^r - 1)\Lambda^r \log \Lambda}{(\Lambda^r - 1)^2 \rho \Lambda \log \Lambda}, & \text{if } \rho \geq 1. \end{cases} \quad (17)$$

- The channel gains are assumed to be independent complex Gaussian random variables with zero mean and variance  $\sigma_{k_i}^2$ , i.e.,  $\alpha_k^{(i)} \sim \mathcal{CN}(0, \sigma_{k_i}^2)$ . This assumption leads  $|\alpha_k^{(i)}|$  to be Rayleigh-distributed with parameter  $\sigma_{k_i}^2/2$ . Although channel modeling for UWB systems is still an open issue, this hypothesis, appealing for its analytical tractability, also provides a good approximation for multipath propagation in UWB systems [5].
- The averaged power delay profile (aPDP) is assumed to decay exponentially, as is customarily taken in most UWB channel models [6]. Hence,  $\sigma_{k_i}^2 = \sigma_k^2 \cdot \Lambda^{-\frac{i-1}{L-1}}$ , where  $\Lambda = \sigma_{k_1}^2/\sigma_{k_L}^2$  and  $\sigma_k^2$  depends on the distance between user  $k$  and the base station. It is easy to verify that  $\Lambda = 0$  dB represents the case of flat aPDP.

**Prop. 1** In the asymptotic case where  $K$  and  $N_f$  are finite, while  $L, N_c \rightarrow \infty$ , when adopting a PRake with  $L_P$  coefficients according to the MRC scheme, the terms  $\zeta_k^{-1}$  and  $\gamma_{0,k}^{-1}$  converge almost surely (a.s.) to

$$\zeta_k^{-1} \xrightarrow{\text{a.s.}} \frac{K-1}{N} \cdot \mu(\Lambda, r), \quad (14)$$

$$\gamma_{0,k}^{-1} \xrightarrow{\text{a.s.}} \frac{1}{N} \cdot \nu(\Lambda, r, \rho), \quad (15)$$

where  $r \triangleq L_P/L$ ,  $0 < r \leq 1$ , and  $\rho \triangleq N_c/L$ ,  $0 < \rho < \infty$ , are held constant, and

$$\mu(\Lambda, r) = \frac{(\Lambda - 1) \cdot \Lambda^{r-1}}{\Lambda^r - 1}, \quad (16)$$

with  $\nu(\Lambda, r, \rho)$  defined as in (17), shown at the top of the page.

The proof of Prop. 1 has been omitted because of space limitation. It can be found in [7].

Proposition 1 gives accurate approximations for the terms of MAI and SI in the case of PRake receivers at the access point and of exponentially decaying aPDP. Results for more specific scenarios can be derived using particular values of  $\Lambda$

and  $r$ , as shown in [7]. As an example, it is possible to obtain approximations for the MAI and SI arising in the ARake as follows:

$$\mu_A(\Lambda) = \lim_{r \rightarrow 1} \mu(\Lambda, r) = 1, \quad (18)$$

$$\nu_A(\Lambda, \rho) = \lim_{r \rightarrow 1} \nu(\Lambda, r, \rho) = \begin{cases} \frac{2(\Lambda^2 - 1 + \Lambda^\rho - \Lambda^{2-\rho} - 2\Lambda\rho \log \Lambda)}{(\Lambda - 1)^2 \rho \log \Lambda}, & \text{if } \rho \leq 1, \\ \frac{2(\Lambda^2 - 1 - 2\Lambda \log \Lambda)}{(\Lambda - 1)^2 \rho \log \Lambda}, & \text{if } \rho \geq 1. \end{cases} \quad (19)$$

### 3.2. Comments on the Results

Fig. 1 shows the shape of  $\mu(\Lambda, r)$  versus  $r$  for some values of  $\Lambda$ . As can be noticed,  $\mu(\Lambda, r)$  is decreasing as either  $\Lambda$  or  $r$  increases. Keeping  $r$  fixed,  $\mu(\Lambda, r)$  is a decreasing function of  $\Lambda$ , since the neglected paths are weaker as  $\Lambda$  increases. Keeping  $\Lambda$  fixed,  $\mu(\Lambda, r)$  is a decreasing function of  $r$ , since the receiver uses a higher number of coefficients, thus better mitigating the effect of MAI.

Fig. 2 shows the shape of  $\nu(\Lambda, r, \rho)$  versus  $r$  for some values of  $\Lambda$  and  $\rho$ . As can be verified,  $\nu(\Lambda, r, \rho)$  decreases as either  $\rho$  or  $\Lambda$  increases. This dependency of  $\nu(\Lambda, r, \rho)$  wrt  $\rho$  is justified by the higher resistance to multipath due to increasing the length of a single frame [2, 4]. Similarly to  $\mu(\Lambda, r)$ ,  $\nu(\Lambda, r, \rho)$  is a decreasing function of  $\Lambda$  when  $r$  and  $\rho$  are fixed, since the neglected paths are weaker as  $\Lambda$  increases. Taking into account the dependency of  $\nu(\Lambda, r, \rho)$  wrt  $r$ , it can be verified that  $\nu(\Lambda, r, \rho)$  is not monotonically decreasing as  $r$  increases. In other words, an ARake receiver using MRC does not offer the optimum performance in mitigating the effect of SI, but it is outperformed by PRake receivers whose  $r$  decreases as  $\Lambda$  increases. This behavior is due to using MRC, which attempts to gather all the signal energy to maximize the signal-to-noise ratio (SNR) and substantially ignores the SI. In this scenario, a minimum mean square error (MMSE) combining criterion, while more complex, might give a different comparison.

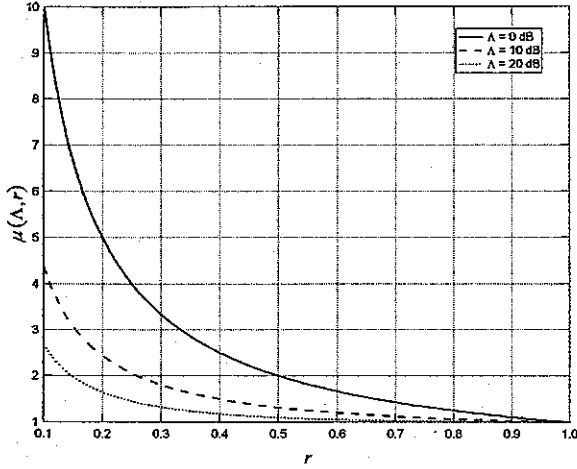


Fig. 1. Shape of  $\mu(\Lambda, r)$  versus  $r$  for some  $\Lambda$ 's.

## 4. ANALYSIS OF THE NASH EQUILIBRIUM

### 4.1. Analytical Results

Using Prop. 1 in (9) and (12), it is straightforward to obtain the utilities  $u_k^*$  at the Nash equilibrium, which are independent of the channel realizations of the other users, and of SI:

$$u_k^* \xrightarrow{a.s.} h_k^{(SP)} \cdot \frac{D}{M} R \frac{f(\bar{\gamma}^*)}{\sigma^2 \bar{\gamma}^*} \times (1 - \bar{\gamma}^* \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]/N). \quad (20)$$

Note that (20) requires the knowledge of the channel realization for user  $k$ . Analogously, (13) translates into

$$N_f \geq \lceil \bar{\gamma}^* \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]/N_c \rceil, \quad (21)$$

where  $\lceil \cdot \rceil$  is the ceiling operator. If (21) does not hold, some users will end up transmitting at maximum power  $\bar{p}$ .

**Prop. 2** *In the asymptotic case where the hypotheses of Prop. 1 hold, the loss  $\Psi = u_{k_A}^*/u_k^*$  of a PRake receiver wrt an ARake receiver in terms of achieved utilities converges a.s. to*

$$\Psi \xrightarrow{a.s.} \mu(\Lambda, r) \cdot \frac{N - \bar{\gamma}^* [(K-1)\mu_A(\Lambda) + \nu_A(\Lambda, \rho)]}{N - \bar{\gamma}^* [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}, \quad (22)$$

where  $u_{k_A}^*$  is the utility achieved by an ARake receiver.

The proof of Prop. 2 can be found in [7]. Equation (22) also provides a system design criterion. Given  $L$ ,  $N_c$ ,  $N_f$ ,  $K$  and  $\Lambda$ , a desired loss  $\Psi$  can in fact be achieved using the ratio  $r$  obtained by numerically inverting (22).

### 4.2. Simulation Results

Simulations are performed using the iterative algorithm described in [2]. We assume that each packet contains 100 b

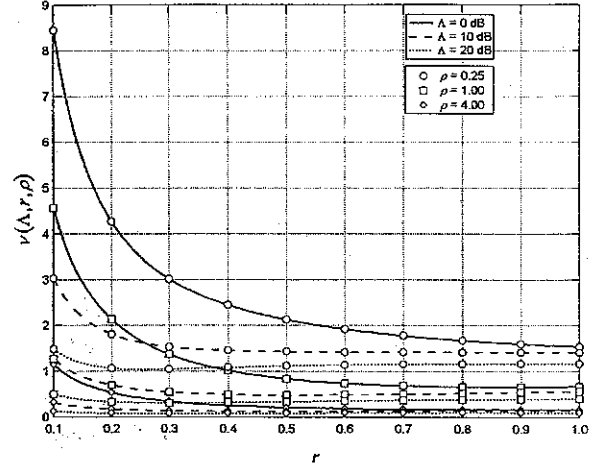


Fig. 2. Shape of  $\nu(\Lambda, r, \rho)$  versus  $r$  for some  $\Lambda$ 's and  $\rho$ 's.

of information and no overhead (i.e.,  $D = M = 100$ ). We use the efficiency function  $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$  as a reasonable approximation to the PSR. Using  $M = 100$ ,  $\bar{\gamma}^* = 11.1$  dB. We also set  $R = 100$  kb/s,  $\sigma^2 = 5 \times 10^{-16}$  W, and  $\bar{p} = 1$   $\mu$ W. To model the UWB scenario, the channel gains are assumed as in Sect. 3, with  $\sigma_k^2 = 0.3d_k^{-2}$ , where  $d_k$  is the distance between the  $k$ th user and the base-station. Distances are assumed to be uniformly distributed between 3 and 20 m.

Fig. 3 shows the probability  $P_o$  of having at least one user transmitting at the maximum power, i.e.,  $P_o = \mathbb{P}\{\max_k p_k = \bar{p} = 1 \mu\text{W}\}$ , as a function of the number of frames  $N_f$ . We consider 10 000 realizations of the channel gains, using a network with  $K = 8$  users,  $N_c = 50$ ,  $L = 200$  (thus  $\rho = 0.25$ ), and PRake receivers with  $L_P = 20$  coefficients (and thus  $r = 0.1$ ). Note that the slope of  $P_o$  increases as  $\Lambda$  increases. This phenomenon is due to reducing the effects of neglected path gains as  $\Lambda$  becomes higher, which, given  $N_f$ , results in having more homogeneous effects of neglected gains. Using the parameters above in (21), the minimum value of  $N_f$  that allows all  $K$  users to simultaneously achieve the optimum SINRs is  $N_f = \{21, 9, 6\}$  for  $\Lambda = \{0 \text{ dB}, 10 \text{ dB}, 20 \text{ dB}\}$ , respectively. As can be seen, the analytical results closely match with simulations. It is worth emphasize that (21) is valid for both  $L$  and  $L_P$  going to  $\infty$ , as stated in Prop. 1. In this example,  $L_P = 20$ , which does not fulfill this hypothesis. This explains the slight mismatch between theoretical and simulation results, especially for small  $\Lambda$ 's. However, the authors have found showing numerical results for a feasible system to be more interesting than simulating a network with a very high number of PRake coefficients.

Fig. 4 shows a comparison between analytical and numerical achieved utilities versus the channel gains  $h_k = \|\alpha_k\|^2$ . The network parameters are  $K = 8$ ,  $L = 200$ ,  $N_c = 50$ ,  $N_f = 20$ ,  $\Lambda = 10$  dB,  $\rho = 0.25$ . The markers correspond to the simulation results given by a single realization of the path

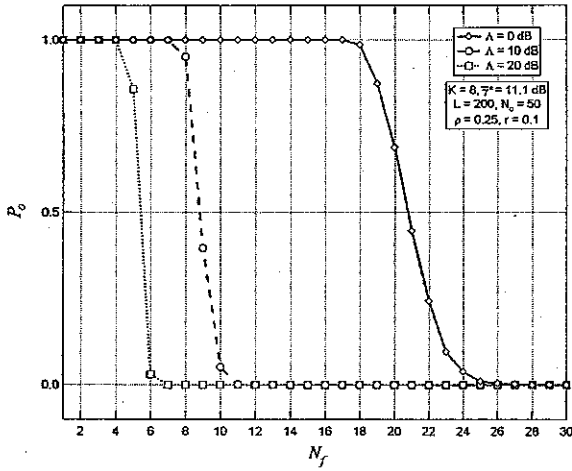


Fig. 3. Probability of having at least one user transmitting at maximum power versus number of frames.

gains. Some values of the receiver coefficients are considered. The solid line represents the theoretical achieved utility, computed using (20) with  $r = 1$ . The dashed, the dash-dotted and the dotted lines have been obtained by subtracting from (20) the loss  $\Psi$ , computed as in (22). Using the parameters above,  $\Psi = \{1.34 \text{ dB}, 2.94 \text{ dB}, 8.40 \text{ dB}\}$  for  $r = \{0.5, 0.3, 0.1\}$ , respectively. It is worth noting that such lines do not consider the effective values of  $h_k^{(SP)}$ , as required in (20),<sup>2</sup> since they make use of the asymptotic approximation (22). The analytical results closely match the actual performance of the PRake receivers, especially recalling that the results are not averaged, but only a single random scenario is considered. As before, the larger the number of  $L_P$  coefficients is, the smaller the difference between theoretical analysis and simulations is.

## 5. CONCLUSION

In this paper, we have used a large system analysis to study performance of PRake receivers using maximal ratio combining schemes when energy-efficient PC techniques are adopted. We have considered a wireless data network in frequency-selective environments, where the user terminals transmit IR-UWB signals to a common concentration point. Assuming the averaged power delay profile and the amplitude of the path coefficients to be exponentially decaying and Rayleigh-distributed, respectively, we have obtained a general characterization for the terms due to self-interference and multiple access interference. The expressions are dependent only on the network parameters and the number of PRake coefficients. A measure of the loss of PRake receivers with respect to the ARake receiver has then been proposed which is completely independent of the channel realizations. This theoretical ap-

<sup>2</sup>This is also valid for the case ARake, since  $h_k^{(SP)} = h_k$ .

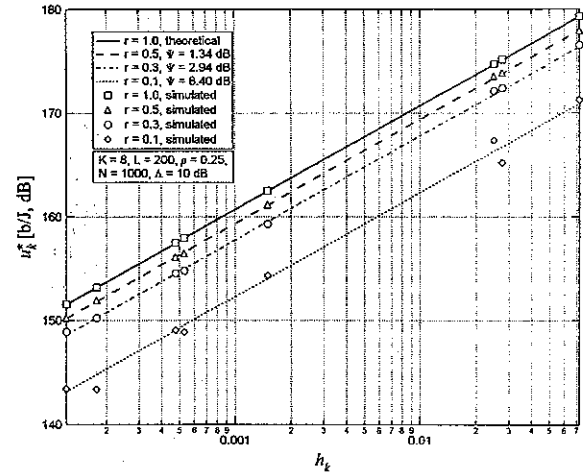


Fig. 4. Achieved utility versus channel gain at the Nash equilibrium for different ratios  $r$ .

proach may also serve as a criterion for network design, since it is completely described by the network parameters.

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# Performance Comparison of Energy-Efficient Power Control for CDMA and Multiuser UWB Networks

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**Abstract**—This paper studies the performance of a wireless data network using energy-efficient power control techniques when different multiple access schemes, namely direct-sequence code division multiple access (DS-CDMA) and impulse-radio ultrawideband (IR-UWB), are considered. Due to the large bandwidth of the system, the multipath channel is assumed to be frequency-selective. By making use of noncooperative game-theoretic models and large-system analysis tools, explicit expressions for the achieved utilities at the Nash equilibrium are derived in terms of the network parameters. A measure of the loss of DS-CDMA with respect to IR-UWB is proposed, which proves substantial equivalence between the two schemes. Simulation results are provided to validate the analysis.

## I. INTRODUCTION

The increasing demand for high-speed data services in wireless networks calls for multiple access schemes with efficient resource allocation and interference mitigation. Both direct-sequence code division multiple access (DS-CDMA) and impulse-radio ultrawideband (IR-UWB) are considered to be potential candidates for such next-generation high-speed networks. Design of reliable systems must include transmitter power control, which aims to allow each mobile terminal to achieve the required quality of service at the uplink receiver while minimizing power consumption. Scalable techniques for energy-efficient power control can be derived using game theory [1]–[3].

This paper compares the performance of game-theoretic power control schemes in the uplink of an infrastructure network using either DS-CDMA or IR-UWB as a multiple access technique. The performance index here is represented by the achieved utility at the Nash equilibrium, where utility is defined as the ratio of the throughput to the transmit power. Due to the large bandwidth occupancy [4], the channel fading is assumed to be frequency-selective. Resorting to a large-system analysis [3], systems with equal spreading factor operating in a dense multipath environment are compared. Both analytical and numerical results show that, even though UWB-based networks always outperform CDMA-based systems, the difference between achieved utilities is so slight that equivalence in terms of energy efficiency can be assumed.

This research was supported in part by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252, and in part by the Network of Excellence in Wireless Communications NEWCOM of the European Commission FP6 under Contract No. 507325.

The remainder of the paper is organized as follows. The system model is described in Sect. II. Sect. III contains the main results of the proposed noncooperative power control game. A comparison between the energy efficiency of the two considered multiple access schemes is performed in Sect. IV, where also simulation results are shown. Some conclusions are drawn in Sect. V.

## II. SYSTEM MODEL

### A. IR-UWB Wireless Networks

We consider the uplink of a binary phase shift keying (BPSK) random time-hopping (TH) IR-UWB system with  $K$  users transmitting to a common concentration point. The transmitted signal from user  $k$  is [5]

$$s_{tx}^{(k)}(t) = \sqrt{\frac{p_k T_f}{N}} \sum_{n=-\infty}^{+\infty} d_n^{(k)} b_{[n/N_f]}^{(k)} w_{tx}(t - nT_f - c_n^{(k)}T_c), \quad (1)$$

where  $w_{tx}(t)$  is the transmitted UWB pulse with duration  $T_c$  and unit energy;  $p_k$  is the transmit power of user  $k$ ;  $T_f$  is the frame time;  $b_{[n/N_f]}^{(k)} \in \{-1, +1\}$  is the information symbol transmitted by user  $k$ ; and  $N = N_f \cdot N_c$  is the processing gain, where  $N_f$  is the number of pulses representing one information symbol, and  $N_c = T_f/T_c$  denotes the number of possible pulse positions in a frame. Throughout this analysis, a system with polarity code randomization is considered [6]. In particular, the polarity code for user  $k$  is  $\mathbf{d}_k = \{d_0^{(k)}, \dots, d_{N_f-1}^{(k)}\}$ , where the  $d_j^{(k)}$ 's are independent random variables taking values  $\pm 1$  with probability  $1/2$ . To allow the channel to be shared by many users without causing catastrophic collisions, a TH sequence  $\mathbf{c}_k = \{c_1^{(k)}, \dots, c_{N_f}^{(k)}\}$  is assigned to each user, where  $c_n^{(k)} \in \{0, 1, \dots, N_c - 1\}$  with equal probability  $1/N_c$ .

Defining a sequence  $\{s_n^{(k)}\}$  as

$$s_n^{(k)} = \begin{cases} d_{[n/N_c]}^{(k)}, & c_{[n/N_c] \cdot N_c}^{(k)} = n - [n/N_c] \cdot N_c, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

we can express (1) as

$$s_{tx}^{(k)}(t) = \sqrt{\frac{p_k T_f}{N}} \sum_{n=-\infty}^{+\infty} s_n^{(k)} b_{[n/N]}^{(k)} w_{tx}(t - nT_c). \quad (3)$$

It is worth noting that this system makes use of a ternary sequence  $\{-1, 0, +1\}$ , where also the elements are dependent, due to the TH sequence.

The transmission is assumed to be over *frequency-selective channels*, with the channel for user  $k$  modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^L \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \quad (4)$$

where  $L$  is the number of channel paths;  $\alpha_k = [\alpha_1^{(k)}, \dots, \alpha_L^{(k)}]^T$  and  $\tau_k$  are the fading coefficients and the delay of user  $k$ , respectively. Considering a chip-synchronous scenario, symbols are misaligned by an integer multiple of  $T_c$ :  $\tau_k = \Delta_k T_c$ , for every  $k$ , where  $\Delta_k$  is uniformly distributed in  $\{0, 1, \dots, N-1\}$ . We also assume that channel characteristics remain unchanged over several symbol intervals [5].

Due to the high resolution of UWB signals, multipath channels can have hundreds of multipath components, especially in indoor environments. To mitigate the effect of multipath fading as much as possible, we consider an access point where  $K$  Rake receivers [7] are used.<sup>1</sup> The Rake receiver for user  $k$  is in general composed of  $L$  coefficients, where the vector  $\beta_k = \mathbf{G} \cdot \alpha_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$  represents the combining weights for user  $k$ , and the  $L \times L$  matrix  $\mathbf{G}$  depends on the type of Rake receiver employed.

The signal-to-interference-plus-noise ratio (SINR) of the  $k$ th user at the output of the Rake receiver can be well approximated (for large  $N_f$ , typically at least 5) by [5]

$$\gamma_k = \frac{h_k^{(\text{SP})} p_k}{h_k^{(\text{SI})} p_k + \sum_{j=1, j \neq k}^K h_{kj}^{(\text{MAI})} p_j + \sigma^2}, \quad (5)$$

where  $\sigma^2$  is the variance of the additive white Gaussian noise (AWGN); and the terms due to signal part (SP), self-interference (SI), and multiple access interference (MAI), are

$$h_k^{(\text{SP})} = \beta_k^H \cdot \alpha_k, \quad (6)$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2}{\beta_k^H \cdot \alpha_k}, \quad (7)$$

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\|\mathbf{B}_k^H \cdot \alpha_j\|^2 + \|\mathbf{A}_j^H \cdot \beta_k\|^2 + |\beta_k^H \cdot \alpha_j|^2}{\beta_k^H \cdot \alpha_k}, \quad (8)$$

respectively, where the matrices

$$\mathbf{A}_k = \begin{pmatrix} \alpha_L^{(k)} & \dots & \dots & \alpha_2^{(k)} \\ 0 & \alpha_L^{(k)} & \dots & \alpha_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \alpha_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (9)$$

<sup>1</sup>For ease of calculation, perfect channel estimation is considered throughout the paper.

$$\mathbf{B}_k = \begin{pmatrix} \beta_L^{(k)} & \dots & \dots & \beta_2^{(k)} \\ 0 & \beta_L^{(k)} & \dots & \beta_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \beta_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (10)$$

$$\Phi = \text{diag}\{\phi_1, \dots, \phi_{L-1}\}, \text{ and } \phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}}, \quad (11)$$

have been introduced for convenience of notation.

### B. DS-CDMA Wireless networks

In order to perform a fair comparison, the uplink of a random DS-CDMA system with spreading factor  $N$  and  $K$  users is considered. It can be noticed that (3) can represent a DS-CDMA system with processing gain  $N$  by considering the special case when  $T_f = T_c$  (and thus  $N_c = 1$ ) [5]. As is apparent from (2), using  $N_c = 1$  yields the elements of  $\{s_n^{(k)}\}$  to be binary independent and identical distributed (i.i.d.).

Hence, in a dense frequency-selective multipath environment, the SINR of user  $k$  at the output of the Rake receiver is also represented by (5), under the conditions  $N_c = 1$ ,  $N = N_f$ .

### III. THE NONCOOPERATIVE POWER CONTROL GAME

Consider now the application of noncooperative power control techniques to the wireless networks described above. Focusing on mobile terminals, where it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput, a game-theoretic energy-efficient approach like the one described in [3] can be considered.

#### A. Analysis of the Nash equilibrium

It is possible to formulate a noncooperative power control game in which each user seeks to maximize its own utility function. Let  $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$  be the proposed noncooperative game where  $\mathcal{K} = \{1, \dots, K\}$  is the index set for the users;  $P_k = [\underline{p}_k, \bar{p}_k]$  is the strategy set, with  $\underline{p}_k$  and  $\bar{p}_k$  denoting minimum and maximum power constraints, respectively; and  $u_k(\mathbf{p})$  is the payoff function for user  $k$  [2], defined as

$$u_k(\mathbf{p}) = \frac{D}{M} R_k \frac{f(\gamma_k)}{p_k}, \quad (12)$$

where  $\mathbf{p} = [p_1, \dots, p_K]$  is the vector of transmit powers;  $D$  and  $M$  are the number of information bits and the total number of bits in a packet, respectively;  $R_k$  and  $\gamma_k$  are the transmission rate and the SINR for the  $k$ th user, respectively; and  $f(\gamma_k)$  is the efficiency function representing the packet success rate (PSR), i.e., the probability that a packet is received without an error. Throughout this analysis, we assume  $\underline{p}_k = 0$  and  $\bar{p}_k = \bar{p}$  for all  $k \in \mathcal{K}$ .

Provided that the efficiency function is increasing, S-shaped, and continuously differentiable, with  $f(0) = 0$ ,  $f(+\infty) = 1$ , and  $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$ , the solution of the

$$\nu(\Lambda, r, \rho) = \begin{cases} \frac{\Lambda(\Lambda^\rho - 1)(4\Lambda^{2r} + 3\Lambda^\rho - 1) - 2\Lambda^{r+\rho}(\Lambda^r + 3\Lambda - 1)\rho \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } 0 \leq \rho \leq \min(r, 1-r); \quad (19a) \\ \frac{\Lambda(4\Lambda^\rho - 1)(\Lambda^{2r} - 1) - 2\Lambda^{r+\rho}(3\Lambda^r - \rho + \Lambda^r \rho) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } r \leq \rho \leq 1-r \text{ and} \\ & r \leq 1/2; \quad (19b) \\ \frac{-4\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} + 3\Lambda^{2+2\rho} - 2\Lambda^{1+r+\rho}(\tau + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } 1-r \leq \rho \leq r \text{ and} \\ & r \geq 1/2; \quad (19c) \\ \frac{-\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} - 2\Lambda^{1+r+\rho}(\tau + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } \max(r, 1-r) \leq \rho \leq 1; \quad (19d) \\ \frac{2\Lambda(\Lambda^{2r} - 1) - (\Lambda^r + r + 3\Lambda r - 1)\Lambda^r \log \Lambda}{(\Lambda^r - 1)^2 \rho \Lambda \log \Lambda}, & \text{if } \rho \geq 1. \quad (19e) \end{cases}$$

maximization problem  $\max_{p_k \in P_k} u_k(\mathbf{p})$  for  $k = 1, \dots, K$  is [3]

$$p_k^* = \min \left\{ \frac{\gamma_k^* \left( \sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^* / \gamma_{0,k})}, \bar{p} \right\}, \quad (13)$$

where

$$\gamma_{0,k} = \frac{h_k^{(\text{SP})}}{h_k^{(\text{SI})}} = N \cdot \frac{(\beta_k^H \cdot \alpha_k)^2}{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2} \geq 1 \quad (14)$$

and  $\gamma_k^*$  is the solution of

$$f'(\gamma_k^*) \gamma_k^* (1 - \gamma_k^* / \gamma_{0,k}) = f(\gamma_k^*), \quad (15)$$

where  $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma_k^*}$ . Since  $\gamma_k^*$  depends only on  $\gamma_{0,k}$ , for convenience of notation a function  $\Gamma(\cdot)$  is defined such that  $\gamma_k^* = \Gamma(\gamma_{0,k})$  [3].

### B. Large-System Analysis

As can be verified in (13), the amount of transmit power  $p_k^*$  required to achieve the target SINR  $\gamma_k^*$  will depend not only on  $h_k^{(\text{SP})}$ , but also on the SI term  $h_k^{(\text{SI})}$  (through  $\gamma_{0,k}$ ) and the MAI (through  $h_{kj}^{(\text{MAI})}$ ). To derive quantitative results for the transmit powers independent of SI and MAI terms, it is possible to resort to the large-system analysis described in [3].

For ease of calculation, the expressions derived in the remainder of the paper consider the following assumptions:

- The channel gains are assumed to be independent complex Gaussian random variables with zero means and variances  $\sigma_{k_i}^2$ , i.e.,  $\alpha_k^{(i)} \sim \mathcal{CN}(0, \sigma_{k_i}^2)$ . This assumption leads  $|\alpha_k^{(i)}|$  to be Rayleigh-distributed with parameter  $\sigma_{k_i}^2/2$ . Although channel modeling for wideband systems is still an open issue, this hypothesis, appealing for its analytical tractability, also provides a good approximation for multipath propagation in UWB systems [8].
- The average power delay profile (aPDP) [9] is assumed to decay exponentially, as is customary used in many UWB channel models [4]. This translates into the hypothesis  $\sigma_{k_i}^2 = \sigma_k^2 \cdot \Lambda^{-\frac{i-1}{L-1}}$ , where  $\Lambda = \sigma_{k_1}^2 / \sigma_{k_L}^2$  and  $\sigma_k^2$  depends on the distance between user  $k$  and the access point.
- Partial-Rake (PRake) receivers with  $L_P$  fingers using maximal ratio combining (MRC) are implemented at the

access point. In other words, we consider  $\mathbf{G}$  to be a deterministic diagonal matrix, with

$$\{\mathbf{G}\}_{ll} = \begin{cases} 1, & 1 \leq l \leq r \cdot L, \\ 0, & \text{elsewhere,} \end{cases} \quad (16)$$

where  $r \triangleq L_P/L$  and  $0 < r \leq 1$ . It is worth noting that, when  $r = 1$ , an all-Rake (ARake) is implemented.

- As is typical in multiuser wideband systems, the number of users is much smaller than the processing gain, i.e.,  $N \gg K$ . This assumption can also be justified since the analysis is performed for dense multipath environments, as shown in the following.
- The maximum transmit power  $\bar{p}$  is assumed to be sufficiently large.

Under the above hypotheses, a large-system analysis can be performed considering a dense multipath environment, with  $L \rightarrow \infty$ . It turns out that the achieved utilities  $u_k^*$  at the Nash equilibrium converge almost surely (a.s.) to [3], [10]

$$u_k^* \xrightarrow{\text{a.s.}} h_k^{(\text{SP})} \cdot \frac{D}{M} R_k \frac{f\left(\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)\right)}{\sigma^2 \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)} \times \left(1 - \frac{\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}{N}\right), \quad (17)$$

where  $r \triangleq L_P/L$ , with  $0 < r \leq 1$ ;  $\rho \triangleq N_c/L$ , with  $0 < \rho < \infty$ ;

$$\mu(\Lambda, r) = \frac{(\Lambda - 1) \cdot \Lambda^{r-1}}{\Lambda^r - 1}; \quad (18)$$

and  $\nu(\Lambda, r, \rho)$  is defined as in (19a)-(19e), shown at the top of the page.

## IV. PERFORMANCE COMPARISON

### A. Analytical Results

The results derived in the previous section allow the performance of IR-UWB and DS-CDMA systems to be compared in terms of achieved utilities at the Nash equilibrium.

For an IR-UWB system, the utility  $u_{k_U}^*$  can be evaluated using (17). In the case of a DS-CDMA system, (17) can still

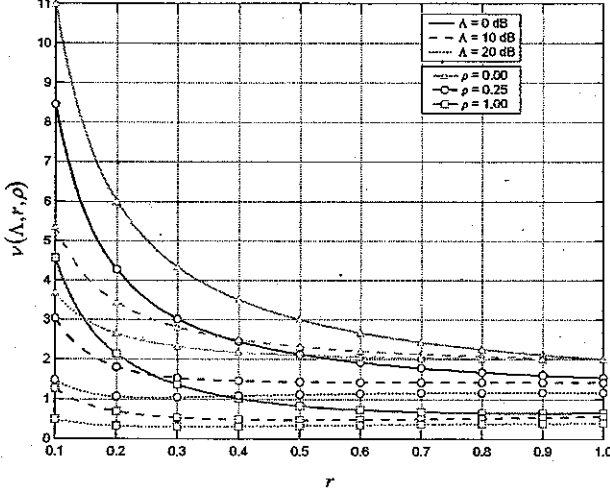


Fig. 1. Shape of  $\nu(\Lambda, r, \rho)$  versus  $r$  for some values of  $\Lambda$  and  $\rho$ .

give the utility  $u_{kC}^*$ , provided that  $\nu(\Lambda, r, \rho)$  is replaced with  $\nu_0(\Lambda, r)$ , defined as

$$\nu_0(\Lambda, r) = \lim_{\rho \rightarrow 0} \nu(\Lambda, r, \rho) = \frac{\Lambda + \Lambda^r - 2\Lambda^{1+r}}{\Lambda - \Lambda^{1+r}}. \quad (20)$$

This result is obtained letting  $\rho$  go to 0 in (19a). The proof, omitted because of space limitation, can be derived using the theorems presented in [10] with  $N_c = 1$ .

Fig. 1 shows the shape of  $\nu(\Lambda, r, \rho)$  as a function of  $r$  for some values of  $\Lambda$  and  $\rho$ . With a slight abuse of notation,  $\nu_0(\Lambda, r)$  is reported as  $\nu(\Lambda, r, 0)$  (triangular markers), while circles and square markers depict  $\rho = 0.25$  and  $\rho = 1.0$ , respectively. As can be noted,  $\nu_0(\Lambda, r) > \nu(\Lambda, r, \rho_1) > \nu(\Lambda, r, \rho_2)$  for any  $\rho_2 > \rho_1 > 0$ . This result is justified by the higher resistance to multipath due to increasing the length of a single frame [3], [5]. Furthermore, keeping  $\rho$  fixed,  $\nu(\Lambda, r, \rho)$  decreases both as  $\Lambda$  and as  $r$  increases. The first behavior makes sense, since the effect of multipath (and thus of SI) is higher in channels with lower  $\Lambda$ . The second behavior reflects the fact that exploiting the diversity by adding a higher number of fingers (and thus increasing  $r$ ) results in better mitigating the frequency-selective fading.

**Proposition 1:** When  $L \rightarrow \infty$ , the loss  $\Phi$  of a CDMA system with respect to (wrt) an IR-UWB scheme with  $N_c$  possible pulse positions converges a.s. to

$$\Phi \triangleq 10 \log_{10}(u_{kU}^*/u_{kC}^*) \xrightarrow{a.s.} (10 \log_{10} e) \cdot \varphi \quad [\text{dB}] \quad (21)$$

where

$$\varphi \triangleq \frac{\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \Delta\nu(\Lambda, r, \rho)}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}, \quad (22)$$

with  $\Delta\nu(\Lambda, r, \rho) = \nu_0(\Lambda, r) - \nu(\Lambda, r, \rho)$ .

**Proof:** Recalling (15), it can be noted that the slope of  $\Gamma(\gamma_{0,k})$  is very small for large values of  $\gamma_{0,k}$ . Using

TABLE I

LIST OF PARAMETERS USED IN THE SIMULATIONS.

$M$ , total number of bits per packet	100 b
$D$ , number of information bits per packet	100 b
$R_k = R$ , bit rate	100 kb/s
$\sigma^2$ , AWGN power at the receiver	$5 \times 10^{-16}$ W
$\bar{p}$ , maximum power constraint	1 $\mu$ W

the hypothesis  $N \gg K > 1$ , a good approximation for  $\Gamma(N/\nu_0(\Lambda, r))$  is  $\Gamma(N/\nu(\Lambda, r, \rho))$ . Therefore, using (17),

$$\frac{u_{kU}^*}{u_{kC}^*} \approx \frac{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot [(K-1)\mu(\Lambda, r) + \nu_0(\Lambda, r)]} \quad (23)$$

$$= \frac{1}{1 - \varphi}, \quad (24)$$

with  $\varphi$  defined as in (22). Recalling that  $N \gg 1$ , it is easy to verify that  $\varphi \ll 1$ . Hence, using a first-order Taylor series approximation, the result (21) is straightforward. ■

As already specified (see also Fig. 1),  $\Delta\nu(\Lambda, r, \rho) > 0$  for any  $\rho > 0$ . Proposition 1 thus states that, using an equal spreading factor in the same multipath scenario, any UWB system outperforms the corresponding CDMA schemes.

Nevertheless, typical values of the network parameters yield very small values of  $\Phi$ , especially as  $N$  increases.<sup>2</sup> Hence, using game-theoretic power control techniques, performance of the two multiple access schemes is practically equivalent.

The validity of these claims is verified in the next subsection using numerical simulations.

### B. Numerical Results

Simulations are performed using the iterative algorithm described in detail in [3]. The systems we examine have the design parameters listed in Table I. We use the efficiency function  $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$  as a reasonable approximation to the PSR [2], [5]. To model the UWB scenario, the channel gains are assumed as in Sect. III, with  $\sigma_k^2 = 0.3d_k^{-2}$ , where  $d_k$  is the distance between user  $k$  and the access point. Distances are assumed to be uniformly distributed between 3 and 30 m.

Fig. 2 shows a comparison between analytical and simulated normalized utilities  $u_k^*/h_k^{(\text{SP})}$  at the Nash equilibrium as a function of the spreading factor  $N$ . A network with  $K = 10$  users is considered, while the aPDP is assumed to be exponentially decaying with  $\Lambda = 20$  dB. The number of paths is  $L = 200$ , thus satisfying the large-system assumption. Red (light) and blue (dark) colors depict the cases ARake ( $r = 1$ ) and PRake ( $r = 0.2$ ), respectively. Lines represent theoretical results provided by (17). In particular, solid lines show analytical values for DS-CDMA ( $N_c = 1$ ), while dashed and dotted lines report the IR-UWB scenario, with  $N_c = 10$  and  $N_c = 50$ , respectively. The markers show the simulation results averaged over 10 000 network realizations. It can be

<sup>2</sup>As expected, larger spreading factors better mitigate multipath effects.



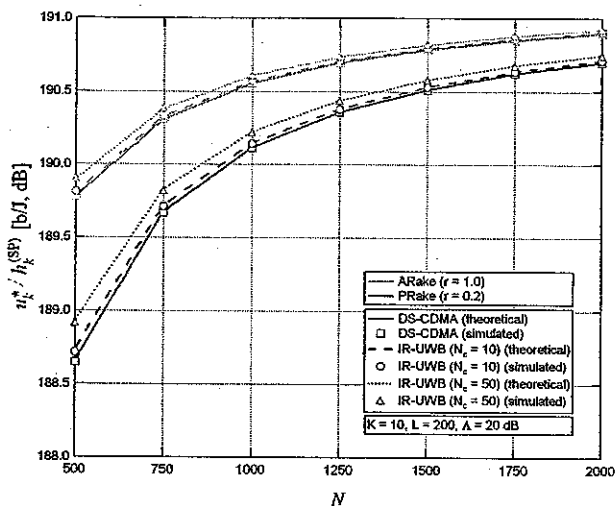


Fig. 2. Comparison of normalized utilities versus processing gain for DS-CDMA and IR-UWB schemes.

seen that the analytical results perfectly match the actual performance of systems. As expected, the performance loss of DS-CDMA wrt IR-UWB is negligible (less than 1 dB) when compared with the normalized achieved utilities. Furthermore, with  $N$  fixed, numerical results confirm that a higher  $r$  provides smaller difference in performance between the two multiple access schemes. Moreover, such loss decreases as  $N$  increases, due to the inherent resistance to multipath, thus smoothing the performance behavior.

Similar considerations can be made when observing the results shown in Fig. 3. The loss of a DS-CDMA wrt an IR-UWB with  $N_c = 50$  is studied. The decay constant of the channel is assumed to be  $\Lambda = 20$  dB. For the sake of presentation, only analytical results are reported. Red (light) and blue (dark) colors represent  $K = 10$  and  $K = 20$ , respectively. The solid lines depict the case ARake, while the dashed lines show the case PRake ( $r = 0.2$ ). The square markers and the circles report the results with  $L = 200$  and  $L = 500$  multiple paths, respectively. It can be seen that the loss  $\Phi$  is always very small. In addition, it is worth noting that, for both  $N$  and  $N_c$  fixed,  $\Phi$  decreases as  $L$  increases. This can be justified since IR-UWB access scheme cannot further mitigate the effects of denser and denser multipath in a  $N_c$ -fixed scenario, and thus its behavior is more similar to that of DS-CDMA systems. This statement is not in contrast with the comments on Fig. 2, since the comparison would be completely different if we considered IR-UWB with fixed  $N$  and variable  $N_c$ . In fact, if we choose  $N_c$  such that  $\rho$  is constant accordingly to the increasing  $L$ ,  $\Phi$  remains unchanged, as is apparent from (21).

## V. CONCLUSION

In this paper, two multiple access schemes, namely DS-CDMA and IR-UWB, have been compared in the context of game-theoretic energy-efficient power control. We have used

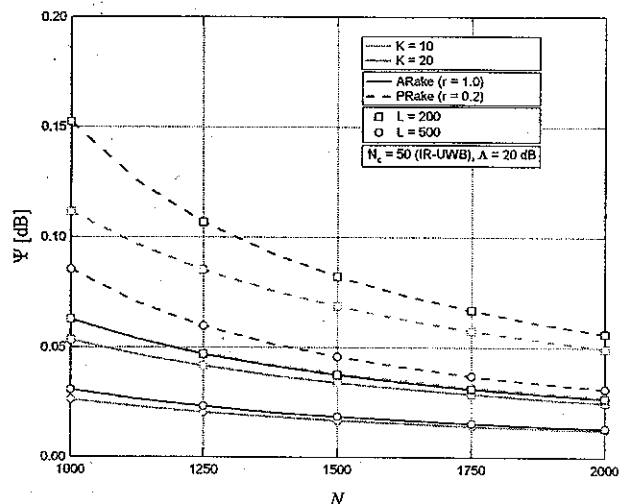


Fig. 3. Performance loss of DS-CDMA wrt IR-UWB for different values of the system parameters.

a large-system analysis to study the performance of a wireless data network using Rake receivers at the access point in a frequency-selective fading channel. Considering systems with equal spreading factor, a measure of the loss of DS-CDMA scheme with respect to IR-UWB multiple access technique has been derived which is dependent only on the network parameters. Theoretical analysis, supported by experimental results, shows that the considered multiple access schemes are practically equivalent in terms of energy efficiency.

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# Performance of Rake Receivers in IR-UWB Networks Using Energy-Efficient Power Control

Giacomo Bacci, Marco Luise and H. Vincent Poor

## Abstract

This paper studies the performance of partial-Rake (PRake) receivers in impulse-radio ultrawideband wireless networks when an energy-efficient power control scheme is adopted. Due to the large bandwidth of the system, the multipath channel is assumed to be frequency-selective. By making use of noncooperative game-theoretic models and large-system analysis tools, explicit expressions are derived in terms of network parameters to measure the effects of self-interference and multiple-access interference at a receiving access point. Performance of the PRake receivers is thus compared in terms of achieved utilities and loss to that of the all-Rake receiver. Simulation results are provided to validate the analysis.

## Index Terms

Energy-efficiency, impulse-radio, ultrawideband systems, Rake receivers, large-system analysis.

## I. INTRODUCTION

Ultrawideband (UWB) technology is considered to be a potential candidate for next-generation multiuser data networks, due to its large spreading factor (which implies large multiuser capacity) and its lower spectral density (which allows coexistence with incumbent systems in the same frequency bands). The requirements for designing high-speed wireless data terminals include efficient resource allocation and interference reduction. These issues aim to allow each user to achieve the required quality of service (QoS) at the uplink receiver without causing unnecessary interference to other users in the system, and minimizing power consumption. Energy-efficient power control techniques can be derived making use of

This research was supported in part by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252, and in part by the U. S. Defense Advanced Research Projects Agency under Grant HR0011-06-1-0052.

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game theory [1]–[6]. In [1], the authors provide motivations for using game theory to study power control in communication systems and ad-hoc networks. In [2], power control is modeled as a noncooperative game in which the users choose their transmit powers to maximize their utilities, defined as the ratio of throughput to transmit power. In [3], [4], the authors use pricing to obtain a more efficient solution for the power control game, while the cross-layer problem of joint multiuser detection and power control is studied in [5]. A game-theoretic approach for a UWB system is studied in [6], where the channel fading is assumed to be frequency-selective, due to the large bandwidth occupancy [7]–[9].

This work extends the results of [6], where a theoretical method to analyze transmit powers and utilities achieved in the uplink of an infrastructure network at the Nash equilibrium has been proposed. However, explicit expressions have been derived in [6] only for all-Rake (ARake) receivers [10] at the access point, under the assumption of a flat averaged power delay profile (aPDP) [11]. This paper considers partial-Rake (PRake) receivers at the access point and makes milder hypotheses on the channel model. Resorting to a large-system analysis, we obtain a general characterization of the effects of multiple access interference (MAI) and self-interference (SI), which allows explicit expressions for the utilities achieved at the Nash equilibrium to be derived. Furthermore, we obtain an approximation to the loss of the PRake receivers with respect to (wrt) the ARake receivers in terms of energy-efficiency, which involves only network parameters and receiver characteristics. Since this loss is independent of the channel realizations, it can serve as a network design criterion.

The remainder of the paper is organized as follows. Some background for this work is given in Sect. II, where the system model is described (Sect. II-A) and the results of the game-theoretic power control approach are shown (Sect. II-B). In Sect. III, we use a large-system analysis to evaluate the effects of the interference at the Nash equilibrium. Results are shown for the general case, as well as for some particular scenarios (including the one proposed in [6]). Performance of the PRake receivers at the Nash equilibrium is analyzed in Sect. IV, where also a comparison with simulation results is provided. Some conclusions are drawn in Sect. V.

## II. BACKGROUND

### A. System Model

Commonly, impulse-radio (IR) systems, which transmit very short pulses with a low duty cycle, are employed to implement UWB systems [12]. We focus on a binary phase shift keying (BPSK) time hopping (TH) IR-UWB system with polarity randomization [13]. A network with  $K$  users transmitting to a receiver at a common concentration point is considered. The processing gain of the system is assumed

to be  $N = N_f \cdot N_c$ , where  $N_f$  is the number of pulses that represent one information symbol, and  $N_c$  denotes the number of possible pulse positions in a frame [12]. The transmission is assumed to be over *frequency selective channels*, with the channel for user  $k$  modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^L \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \quad (1)$$

where  $T_c$  is the duration of the transmitted UWB pulse, which is the minimum resolvable path interval;  $L$  is the number of channel paths;  $\alpha_k = [\alpha_1^{(k)}, \dots, \alpha_L^{(k)}]^T$  and  $\tau_k$  are the fading coefficients and the delay of user  $k$ , respectively. Considering a chip-synchronous scenario, the symbols are misaligned by an integer multiple of the chip interval  $T_c$ :  $\tau_k = \Delta_k T_c$ , for every  $k$ , where  $\Delta_k$  is uniformly distributed in  $\{0, 1, \dots, N-1\}$ . In addition we assume that the channel characteristics remain unchanged over a number of symbol intervals. This can be justified since the symbol duration in a typical application is on the order of tens or hundreds of nanoseconds, and the coherence time of an indoor wireless channel is on the order of tens of milliseconds.

Due to high resolution of UWB signals, multipath channels can have hundreds of multipath components, especially in indoor environments. To mitigate the effect of multipath fading as much as possible, we consider an access point where  $K$  Rake receivers [10] are used.<sup>1</sup> The Rake receiver for user  $k$  is in general composed of  $L$  coefficients, where the vector  $\beta_k = \mathbf{G} \cdot \alpha_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$  represents the combining weights for user  $k$ , and the  $L \times L$  matrix  $\mathbf{G}$  depends on the type of Rake receiver employed. In particular, if  $\mathbf{G}$  is a deterministic diagonal matrix, with

$$\{\mathbf{G}\}_{ll} = \begin{cases} 1, & 1 \leq l \leq r \cdot L, \\ 0, & \text{elsewhere,} \end{cases} \quad (2)$$

where  $r \triangleq L_P/L$  and  $0 < L_P \leq L$ , a PRake with  $L_P$  fingers using maximal ratio combining (MRC) is considered. It is worth noting that, when  $r = 1$ , an ARake is implemented.

The signal-to-interference-plus-noise ratio (SINR) of the  $k$ th user at the output of the Rake receiver can be well approximated<sup>2</sup> by [14]

$$\gamma_k = \frac{h_k^{(\text{SP})} p_k}{h_k^{(\text{SI})} p_k + \sum_{\substack{j=1 \\ j \neq k}}^K h_{kj}^{(\text{MAI})} p_j + \sigma^2}, \quad (3)$$

<sup>1</sup>Since the focus of this work is on the interplay between power control and Rake receivers, perfect channel estimation is considered throughout the paper for ease of calculation.

<sup>2</sup>This approximation is valid for large  $N_f$  (typically, at least 5).

where  $\sigma^2$  is the variance of the additive white Gaussian noise (AWGN) at the receiver, and the gains are expressed by

$$h_k^{(\text{SP})} = \beta_k^H \cdot \alpha_k, \quad (4)$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2}{\beta_k^H \cdot \alpha_k}, \quad (5)$$

and

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\|\mathbf{B}_k^H \cdot \alpha_j\|^2 + \|\mathbf{A}_j^H \cdot \beta_k\|^2 + |\beta_k^H \cdot \alpha_j|^2}{\beta_k^H \cdot \alpha_k}, \quad (6)$$

where

$$\mathbf{A}_k = \begin{pmatrix} \alpha_L^{(k)} & \dots & \dots & \alpha_2^{(k)} \\ 0 & \alpha_L^{(k)} & \dots & \alpha_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \alpha_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (7)$$

$$\mathbf{B}_k = \begin{pmatrix} \beta_L^{(k)} & \dots & \dots & \beta_2^{(k)} \\ 0 & \beta_L^{(k)} & \dots & \beta_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \beta_L^{(k)} \\ 0 & \dots & \dots & 0 \end{pmatrix}, \quad (8)$$

$$\Phi = \text{diag}\{\phi_1, \dots, \phi_{L-1}\}, \quad (9)$$

and

$$\phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}} \quad (10)$$

have been introduced for convenience of notation.

### B. The Game-Theoretic Power Control Game

Consider the application of noncooperative power control techniques to the wireless network described above. Focusing on mobile terminals, where it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput, an energy-efficient approach like the one described in [6] is considered.

Game theory [1] is the natural framework for modeling and studying these interactions between users. It is thus possible to consider a noncooperative power control game in which each user seeks to maximize its own utility function as follows. Let  $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$  be the proposed noncooperative game where  $\mathcal{K} = \{1, \dots, K\}$  is the index set for the users;  $P_k = [\underline{p}_k, \bar{p}_k]$  is the strategy set, with  $\underline{p}_k$  and  $\bar{p}_k$  denoting minimum and maximum power constraints, respectively; and  $u_k(\mathbf{p})$  is the payoff function for user  $k$  [4], defined as

$$u_k(\mathbf{p}) = \frac{D}{M} R_k \frac{f(\gamma_k)}{p_k}, \quad (11)$$

where  $\mathbf{p} = [p_1, \dots, p_K]$  is the vector of transmit powers;  $D$  and  $M$  are the number of information bits per packet and the total number of bits per packet, respectively;  $R_k$  and  $\gamma_k$  are the transmission rate and the SINR (3) for the  $k$ th user, respectively; and  $f(\gamma_k)$  is the efficiency function representing the packet success rate (PSR), i.e., the probability that a packet is received without an error. Throughout this analysis, we assume  $\underline{p}_k = 0$  and  $\bar{p}_k = \bar{p}$  for all  $k \in \mathcal{K}$ .

Provided that the efficiency function is increasing, S-shaped, and continuously differentiable, with  $f(0) = 0$ ,  $f(+\infty) = 1$ , and  $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$ , it has been shown [6] that the solution of the maximization problem  $\max_{p_k \in P_k} u_k(\mathbf{p})$  for  $k = 1, \dots, K$  is

$$p_k^* = \min \left\{ \frac{\gamma_k^* \left( \sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^*/\gamma_{0,k})}, \bar{p} \right\}, \quad (12)$$

where

$$\gamma_{0,k} = \frac{h_k^{(\text{SP})}}{h_k^{(\text{SI})}} = N \cdot \frac{(\beta_k^H \cdot \alpha_k)^2}{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2} \geq 1 \quad (13)$$

and  $\gamma_k^*$  is the solution of

$$f'(\gamma_k^*) \gamma_k^* (1 - \gamma_k^*/\gamma_{0,k}) = f(\gamma_k^*), \quad (14)$$

where  $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma_k^*}$ . Since  $\gamma_k^*$  depends only on  $\gamma_{0,k}$ , for convenience of notation a function  $\Gamma(\cdot)$  is defined such that  $\gamma_k^* = \Gamma(\gamma_{0,k})$ . Fig. 1 shows the shape of  $\gamma_k^* = \Gamma(\gamma_{0,k})$ , where the efficiency function is taken to be  $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$ , with  $M = 100$ .

Assuming the typical case of multiuser UWB systems, where  $N \gg K$ , and also considering  $\bar{p}$  sufficiently large, (12) can be reduced to [6]

$$p_k^* = \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \Gamma(\gamma_{0,k})}{1 - \Gamma(\gamma_{0,k}) \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1})}, \quad (15)$$

where  $\zeta_k^{-1} = \sum_{j \neq k} h_{kj}^{(\text{MAI})}/h_j^{(\text{SP})}$ ; and  $\gamma_{0,k}^{-1}$  is defined as in (13).

A necessary and sufficient condition for the Nash equilibrium to be achieved simultaneously by all  $K$  users, and thus for (15) to be valid, is [6]

$$\Gamma(\gamma_{0,k}) \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1}) < 1 \quad \forall k \in \mathcal{K}. \quad (16)$$

As can be verified, the amount of transmit power  $p_k^*$  required to achieve the target SINR  $\gamma_k^*$  will depend not only on the gain  $h_k^{(\text{SP})}$ , but also on the SI term  $h_k^{(\text{SI})}$  (through  $\gamma_{0,k}$ ) and the interferers  $h_{kj}^{(\text{MAI})}$  (through  $\zeta_k$ ).

### III. ANALYSIS OF THE INTERFERENCE

In order to derive some quantitative results for the achieved utilities and for the transmit powers independent of SI and MAI terms, it is possible to resort to a large-system analysis.

*Theorem 1 ([6]):* Assume that  $\alpha_k^{(l)}$  are zero-mean random variables independent across  $k$  and  $l$ , and  $\mathbf{G}$  is a deterministic diagonal matrix (thus implying that  $\alpha_k^{(l)}$  and  $\beta_j^{(m)}$  are dependent only when  $j = k$  and  $m = l$ ). In the asymptotic case where  $K$  and  $N_f$  are finite,<sup>3</sup> while  $L, N_c \rightarrow \infty$ , with the ratio  $N_c/L$  approaching a constant, the terms  $\zeta_k^{-1}$  and  $\gamma_{0,k}^{-1}$  converge almost surely (a.s.) to

$$\zeta_k^{-1} \xrightarrow{\text{a.s.}} \frac{1}{N} \sum_{\substack{j=1 \\ j \neq k}}^K \frac{\varphi(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha) + \varphi(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta)}{\varphi(\mathbf{D}_j^\alpha \mathbf{D}_j^\beta) \cdot \varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta)} \quad (17)$$

and

$$\gamma_{0,k}^{-1} \xrightarrow{\text{a.s.}} \frac{1}{N} \frac{\lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L+l-i)}{(\varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta))^2}, \quad (18)$$

where  $\phi_i$  is defined as in (10);  $\mathbf{D}_j^\alpha$  and  $\mathbf{D}_k^\beta$  are diagonal matrices whose elements are

$$\{\mathbf{D}_j^\alpha\}_l = \sqrt{\text{Var}[\alpha_j^{(l)}]}, \quad (19)$$

and

$$\{\mathbf{D}_k^\beta\}_l = \sqrt{\text{Var}[\beta_k^{(l)}]}, \quad (20)$$

<sup>3</sup>In order for the analysis to be consistent, and also considering regulations by the US Federal Communications Commission (FCC) [15], it is worth noting that  $N_f$  could not be smaller than a certain threshold ( $N_f \geq 5$ ).

with  $\text{Var}[\cdot]$  denoting the variance of a random variable;  $C_j^\alpha$  and  $C_j^\beta$  are  $L \times (L - 1)$  matrices whose elements are

$$\{C_j^\alpha\}_{li} = \sqrt{\frac{\text{Var}[\{A_j\}_{li}]}{L}}, \quad (21)$$

and

$$\{C_k^\beta\}_{li} = \sqrt{\frac{\text{Var}[\{B_k\}_{li}]}{L}}, \quad (22)$$

$\varphi(\cdot)$  is the matrix operator

$$\varphi(\cdot) = \lim_{L \rightarrow \infty} \frac{1}{L} \text{Tr}(\cdot), \quad (23)$$

with  $\text{Tr}(\cdot)$  denoting the trace operator; and

$$\theta_k(l, L + l - i) = \{D_k^\alpha\}_l \{D_k^\beta\}_{L+l-i} + \{D_k^\beta\}_l \{D_k^\alpha\}_{L+l-i}. \quad (24)$$

The proof of this theorem can be found in [6].

The results above can be applied to any kind of fading model, as long as the second-order statistics are available. Furthermore, due to the symmetry of (17) and (18), it is easy to verify that the results are independent of large-scale fading models. Hence, Theorem 1 applies to any kind of channel, which may include both large- and small-scale statistics.

Channel modeling for IR-UWB systems is still an open issue. In fact, while there exists a commonly agreed-on set of basic models for narrowband and wideband wireless channels [16], a similarly well accepted UWB channel model does not seem to exist. Recently, two models, namely IEEE 802.15.3a [8] and IEEE 802.15.4a [9], have been standardized to properly characterize the UWB environment. However, for ease of calculation, the expressions derived in the remainder of the paper consider the following simplifying assumptions:

- The channel gains are independent complex Gaussian random variables with zero means and variances  $\sigma_{k_l}^2$ , i.e.,  $\alpha_k^{(l)} \sim \mathcal{CN}(0, \sigma_{k_l}^2)$ . This assumption leads  $|\alpha_k^{(l)}|$  to be Rayleigh-distributed with parameter  $\sigma_{k_l}^2/2$ . Although both IEEE 802.15.3a and IEEE 802.15.4a models include some forms of Nakagami  $m$  distribution for the channel gains, the Rayleigh distribution, appealing for its analytical tractability, has recently been shown [17] to provide a good approximation for multipath propagation in UWB systems.
- Lately, a clustering phenomenon for the aPDP [11] in IR-UWB multipath channels has emerged from a large number of UWB measurement campaigns [18], [19]. However, owing to the analytical difficulties arising when considering such aspect, this work focuses on an exponentially decaying



aPDP, as is customarily used in several UWB channel models [20], [21]. This translates into the hypothesis

$$\sigma_{k_l}^2 = \sigma_k^2 \cdot \Lambda^{-\frac{l-1}{L-1}}, \quad (25)$$

where

$$\Lambda = \sigma_{k_1}^2 / \sigma_{k_L}^2 \quad (26)$$

and the variance  $\sigma_k^2$  depends on the distance between user  $k$  and the access point. Fig. 2 shows the aPDP for some values of  $\Lambda$  versus the normalized excess delay, i.e., the ratio between the excess delay,  $lT_c$ , and the maximum excess delay considered,  $LT_c$ . It is easy to verify that  $\Lambda = 0$  dB represents the case of flat aPDP.

Using these hypotheses, the matrices  $\mathbf{D}_k^\alpha$  and  $\mathbf{D}_k^\beta$  can be expressed in terms of

$$\{\mathbf{D}_k^\alpha\}_l = \sigma_k \cdot \Lambda^{-\frac{l-1}{2(L-1)}} \cdot u[L-l] \quad (27)$$

and

$$\{\mathbf{D}_k^\beta\}_l = \sigma_k \cdot \Lambda^{-\frac{l-1}{2(L-1)}} \cdot u[r \cdot L - l], \quad (28)$$

where

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases} \quad (29)$$

#### A. PRake with exponentially decaying aPDP

*Prop. 1:* In the asymptotic case where the hypotheses of Theorem 1 hold, when adopting a PRake with  $L_P$  coefficients according to the MRC scheme,

$$\zeta_k^{-1} \xrightarrow{a.s.} \frac{K-1}{N} \cdot \mu(\Lambda, r), \quad (30)$$

where

$$\mu(\Lambda, r) = \frac{(\Lambda - 1) \cdot \Lambda^{r-1}}{\Lambda^r - 1}, \quad (31)$$

and  $r \triangleq L_P/L$ ,  $0 < r \leq 1$ .

The proof can be found in App. A.

*Prop. 2:* In the asymptotic case where the hypotheses of Theorem 1 hold, when adopting a PRake with  $L_P$  coefficients according to the MRC scheme,

$$\gamma_{0,k}^{-1} \xrightarrow{a.s.} \frac{1}{N} \cdot \nu(\Lambda, r, \rho), \quad (32)$$

where  $\rho \triangleq N_c/L$ ,  $0 < \rho < \infty$ ,  $r \triangleq L_P/L$ ,  $0 < r \leq 1$ , and

$$\nu(\Lambda, r, \rho) = \begin{cases} \frac{\Lambda(\Lambda^\rho - 1)(4\Lambda^{2r} + 3\Lambda^\rho - 1) - 2\Lambda^{r+\rho}(\Lambda^r + 3\Lambda - 1)\rho \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } 0 \leq \rho \leq \min(r, 1 - r); \\ \frac{\Lambda(4\Lambda^\rho - 1)(\Lambda^{2r} - 1) - 2\Lambda^{r+\rho}(3\Lambda^r - \rho + \Lambda^r \rho) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{1+\rho} \log \Lambda}, & \text{if } \min(r, 1 - r) \leq \rho \leq \max(r, 1 - r) \text{ and } r \leq 1/2; \\ \frac{-4\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} + 3\Lambda^{2+2\rho} - 2\Lambda^{1+r+\rho}(r + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } \min(r, 1 - r) \leq \rho \leq \max(r, 1 - r) \text{ and } r \geq 1/2; \\ \frac{-\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho} - 2\Lambda^{1+r+\rho}(r + 3\Lambda\rho + \Lambda^r \rho - 1) \log \Lambda}{2(\Lambda^r - 1)^2 \rho \Lambda^{2+\rho} \log \Lambda}, & \text{if } \max(r, 1 - r) \leq \rho \leq 1; \\ \frac{2\Lambda(\Lambda^{2r} - 1) - (\Lambda^r + r + 3\Lambda r - 1)\Lambda^r \log \Lambda}{(\Lambda^r - 1)^2 \rho \Lambda \log \Lambda}, & \text{if } \rho \geq 1. \end{cases} \quad \begin{matrix} (33a) \\ (33b) \\ (33c) \\ (33d) \\ (33e) \end{matrix}$$

The proof can be found in App. B.

Propositions 1 and 2 give accurate approximations for the MAI and SI terms in the general case of PRake receivers at the access point and of exponentially decaying aPDP. Furthermore, these results confirm that the approximations are independent of large-scale fading models, as claimed in [6], since they do not depend on the variance of the users.

It is also possible to obtain results for more specific scenarios using (30) and (32) with particular values of  $\Lambda$  and  $r$ , as shown in the following subsections.

### B. PRake with flat aPDP

The results presented above can be used to study the case of a channel model assuming flat aPDP. As already mentioned, the flat aPDP model is captured when  $\Lambda = 1$ . In order to obtain expressions suitable for this case, it is sufficient to let  $\Lambda$  go to 1 in both (30) and (32). The former yields

$$\lim_{\Lambda \rightarrow 1} \mu(\Lambda, r) = \frac{1}{r}, \quad (34)$$

while the latter gives

$$\lim_{\Lambda \rightarrow 1} \nu(\Lambda, r, \rho) = \begin{cases} \frac{2r^2 + 2r - 4\rho r + \rho^2}{2r^2}, & \text{if } 0 \leq \rho \leq \min(r, 1 - r); & (35a) \\ \frac{1}{2} \left( \frac{2-\rho}{r} + \frac{r}{\rho} - 1 \right), & \text{if } \min(r, 1 - r) \leq \rho \leq \max(r, 1 - r) \text{ and } r \leq 1/2; & (35b) \\ \frac{r^3 + r^2(9\rho - 3) + r(3 - 9\rho^2) + 4\rho^3 - 3\rho^2 + 3\rho - 1}{6\rho r^2}, & \text{if } \min(r, 1 - r) \leq \rho \leq \max(r, 1 - r) \text{ and } r \geq 1/2; & (35c) \\ \frac{4r^3 - 3r^2 + 3r + (\rho - 1)^3}{6\rho r^2}, & \text{if } \max(r, 1 - r) \leq \rho \leq 1; & (35d) \\ \frac{4r^2 - 3r + 3}{6\rho r}, & \text{if } \rho \geq 1. & (35e) \end{cases}$$

### C. ARake with exponentially decaying aPDP

The results of Props. 1-2 can also describe the model of a wireless network using ARake receivers at the access point. As noticed in Sect. II-A, an ARake receiver is a PRake receiver with  $r = 1$ . Letting  $r$  go to 1 in (30) and (32), it is possible to obtain approximations for the MAI and SI terms in a multipath channel with exponentially decaying aPDP as follows:

$$\begin{aligned} \mu_A(\Lambda) &= \lim_{r \rightarrow 1} \mu(\Lambda, r) = 1, & (36) \\ \nu_A(\Lambda, \rho) &= \lim_{r \rightarrow 1} \nu(\Lambda, r, \rho) = \begin{cases} \frac{2(\Lambda^2 - 1 + \Lambda^\rho - \Lambda^{2-\rho} - 2\Lambda\rho \log \Lambda)}{(\Lambda - 1)^2 \rho \log \Lambda}, & \text{if } \rho \leq 1, \\ \frac{2(\Lambda^2 - 1 - 2\Lambda \log \Lambda)}{(\Lambda - 1)^2 \rho \log \Lambda}, & \text{if } \rho \geq 1. \end{cases} & (37) \end{aligned}$$

It is worth noting that the result for  $\rho \leq 1$  in (37) has been obtained by letting  $r \rightarrow 1$  in (33c).

### D. ARake with flat aPDP

The simplest case is represented by a wireless network using the ARake receivers at the access point, where the channel is assumed to have a flat aPDP. This situation can be captured by simultaneously

letting both  $\Lambda$  and  $r$  go to 1 in (30) and (32). This approach gives

$$\lim_{\substack{\Lambda \rightarrow 1, \\ r \rightarrow 1}} \mu(\Lambda, r) = 1, \quad (38)$$

$$\lim_{\substack{\Lambda \rightarrow 1, \\ r \rightarrow 1}} \nu(\Lambda, r, \rho) = \begin{cases} \frac{2}{3}(\rho^2 - 3\rho + 3), & \text{if } \rho \leq 1, \\ 2/(3\rho), & \text{if } \rho \geq 1. \end{cases} \quad (39)$$

As in (37), the result for  $\rho \leq 1$  in (39) has been obtained by letting  $r \rightarrow 1, \Lambda \rightarrow 1$  in (33c).

It is worth noting that (38) and (39) coincide with the results obtained in [6] for the specific case of ARake receivers and flat aPDP.

### E. Comments on the Results

This subsection contains some comments on the results provided by Props. 1-2, applied both to the general case of the PRake receivers with an exponentially decaying aPDP and to its subcases.

Fig. 3 shows the shape of the term  $\mu(\Lambda, r)$ , proportional to the MAI as in (30), versus the ratio  $r$  for some values of  $\Lambda$ . The solid line represents  $\Lambda = 0$  dB, while the dashed and the dotted line depict  $\Lambda = 10$  dB and  $\Lambda = 20$  dB, respectively. As can be seen,  $\mu(\Lambda, r)$  decreases as either  $\Lambda$  or  $r$  increases. Keeping  $r$  fixed, it makes sense that  $\mu(\Lambda, r)$  is a decreasing function of  $\Lambda$ , since the received power of the other users is lower as  $\Lambda$  increases. Keeping  $\Lambda$  fixed, it makes sense that  $\mu(\Lambda, r)$  is a decreasing function of  $r$ , since the receiver uses a higher number of coefficients, thus better mitigating the effect of MAI. Furthermore, it can be seen that, for an ARake,  $\lim_{r \rightarrow 1} \mu(\Lambda, r) = \mu_A(\Lambda) = 1$  irrespectively of  $\Lambda$ .

Fig. 4 shows the shape of the term  $\nu(\Lambda, r, \rho)$ , proportional to the SI as in (32), versus the ratio  $r$  for some values of  $\Lambda$  and  $\rho$ . The solid line represents  $\Lambda = 0$  dB, while the dashed and the dotted line depict  $\Lambda = 10$  dB and  $\Lambda = 20$  dB, respectively. The circles represent  $\rho = 0.25$ , while the square markers and the rhombi report the shape of  $\nu(\Lambda, r, \rho)$  for  $\rho = 1.0$  and  $\rho = 4.0$ , respectively. As can be verified,  $\nu(\Lambda, r, \rho)$  decreases as either  $\rho$  or  $\Lambda$  increases. This behavior of  $\nu(\Lambda, r, \rho)$  wrt  $\rho$  is justified by the higher resistance to multipath due to increasing the number of possible positions and thus the length of a single frame. This also agrees with the results of [6] and [14], where it has been shown that, for a fixed total processing gain  $N$ , systems with higher  $N_c$  outperform those with smaller  $N_c$ , due to higher mitigation of SI. Similarly to  $\mu(\Lambda, r)$ , it makes sense that  $\nu(\Lambda, r, \rho)$  is a decreasing function of  $\Lambda$  when  $r$  and  $\rho$  are fixed, since the neglected paths are weaker as  $\Lambda$  increases. Taking into account the behavior of  $\nu(\Lambda, r, \rho)$  as a function of  $r$ , it can be verified, either analytically or graphically, that  $\nu(\Lambda, r, \rho)$  is not monotonically decreasing as  $r$  increases. In other words, an ARake receiver using MRC does not offer the

optimum performance in mitigating the effect of SI, but it is outperformed by the PRake receivers whose  $r$  decreases as  $\Lambda$  increases. This behavior is due to the fact that the receiver uses MRC, which attempts to gather all the signal energy to maximize the signal-to-noise ratio (SNR) and substantially ignores the effects of SI [22]. In this scenario, a minimum mean square error (MMSE) combining criterion [23], while more complex, might give a different comparison.

#### IV. ANALYSIS OF THE NASH EQUILIBRIUM

Making use of the analysis presented in the previous section, it is possible to study the performance of the PRake receivers in terms of achieved utilities when the noncooperative power control techniques described in Sect. II-B are adopted.

##### A. Analytical Results

Using Props. 1 and 2 in (11) and (15), it is straightforward to obtain the expressions for transmit powers  $p_k^*$  and utilities  $u_k^*$  achieved at the Nash equilibrium, which are independent of the channel realizations of the other users, and of SI:

$$p_k^* \xrightarrow{\text{a.s.}} \frac{1}{h_k^{(\text{SP})}} \cdot \frac{N\sigma^2\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}, \quad (40)$$

$$u_k^* \xrightarrow{\text{a.s.}} h_k^{(\text{SP})} \cdot \frac{D}{M} R_k \cdot f\left(\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)\right) \cdot \frac{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}{N\sigma^2\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)}. \quad (41)$$

Note that (40)-(41) require knowledge of the channel realization for user  $k$  (through  $h_k^{(\text{SP})}$ ).

Analogously, (16) translates into the system design parameter

$$N_f \geq \left\lceil \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \frac{(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)}{N_c} \right\rceil, \quad (42)$$

where  $\lceil \cdot \rceil$  is the ceiling operator.

*Prop. 3:* In the asymptotic case where the hypotheses of Theorem 1 hold, the loss  $\Psi$  of a PRake receiver wrt an ARake receiver in terms of achieved utilities converges a.s. to

$$\Psi = \frac{u_{k_A}^*}{u_k^*} \xrightarrow{\text{a.s.}} \mu(\Lambda, r) \cdot \frac{f\left(\Gamma\left(\frac{N}{\nu_A(\Lambda, \rho)}\right)\right)}{f\left(\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)\right)} \cdot \frac{\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)}{\Gamma\left(\frac{N}{\nu_A(\Lambda, \rho)}\right)} \cdot \frac{N - \Gamma\left(\frac{N}{\nu_A(\Lambda, \rho)}\right) [(K-1)\mu_A(\Lambda) + \nu_A(\Lambda, \rho)]}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) [(K-1)\mu(\Lambda, r) + \nu(\Lambda, r, \rho)]}, \quad (43)$$

where  $u_{k_A}^*$  is the utility achieved by an ARake receiver.

The proof can be found in App. C.

Equation (43) also provides a system design criterion. Given  $L$ ,  $N_c$ ,  $N_f$ ,  $K$  and  $\Lambda$ , a desired loss  $\Psi$  can in fact be achieved using the ratio  $r$  obtained by numerically inverting (43). Unlike (40)-(41), this result is independent of all channel realizations.

### B. Simulation Results

In this subsection, we show numerical results for the analysis presented in the previous subsection. Simulations are performed using the iterative algorithm described in detail in [6]. The systems we examine have the design parameters listed in Table I. We use the efficiency function  $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$  as a reasonable approximation to the PSR [4], [14]. To model the UWB scenario, the channel gains are assumed as in Sect. III, with  $\sigma_k^2 = 0.3d_k^{-2}$ , where  $d_k$  is the distance between the  $k$ th user and the access point. Distances are assumed to be uniformly distributed between 3 and 20 m.

Fig. 5 shows the probability  $P_o$  of having at least one user transmitting at the maximum power, i.e.,  $P_o = \Pr\{\max_k p_k = \bar{p} = 1 \mu W\}$ , as a function of the number of frames  $N_f$ . We consider 10 000 realizations of the channel gains, using a network with  $K = 8$  users,  $N_c = 50$ ,  $L = 200$  (thus  $\rho = 0.25$ ), and PRake receivers with  $L_P = 20$  coefficients (and thus  $r = 0.1$ ). The solid line represents the case  $\Lambda = 0$  dB, while the dashed and the dotted lines depict the cases  $\Lambda = 10$  dB and  $\Lambda = 20$  dB, respectively. Note that the slope of  $P_o$  increases as  $\Lambda$  increases. This phenomenon is due to reducing the effects of neglected path gains as  $\Lambda$  becomes higher, which, given  $N_f$ , results in having more homogeneous effects of neglected gains. Using the parameters above in (42), the minimum value of  $N_f$  that allows all  $K$  users to simultaneously achieve the optimum SINRs is  $N_f = \{21, 9, 6\}$  for  $\Lambda = \{0 \text{ dB}, 10 \text{ dB}, 20 \text{ dB}\}$ , respectively. As can be seen, the analytical results closely match those from simulations. It is worth emphasizing that (42) is valid for both  $L$  and  $L_P$  going to  $\infty$ , as stated in Props. 1-2. In this example,  $L_P = 20$ , which does not fulfill this hypothesis. This explains the slight mismatch between theoretical and simulation results, especially for small  $\Lambda$ 's. However, showing numerical results for a feasible system is more interesting than simulating a network with a very high number of PRake coefficients.

Fig. 6 shows a comparison between analytical and numerical achieved utilities as a function of the channel gains  $h_k = \|\alpha_k\|^2$ . The network has the following parameters:  $K = 8$ ,  $L = 200$ ,  $N_c = 50$ ,  $N_f = 20$ ,  $\Lambda = 10$  dB,  $\rho = 0.25$ . The markers correspond to the simulation results given by a single realization of the path gains. Some values of the number of coefficients of the PRake receiver are considered. In particular, the square markers report the results for the ARake ( $r = 1$ ), while triangles, circles and rhombi show the cases  $r = \{0.5, 0.3, 0.1\}$ , respectively. The solid line represents the theoretical achieved utility, computed using (41). The dashed, the dash-dotted and the dotted lines have been obtained by subtracting

from (41) the loss  $\Psi$ , computed as in (43). Using the parameters above,  $\Psi = \{1.34 \text{ dB}, 2.94 \text{ dB}, 8.40 \text{ dB}\}$  for  $r = \{0.5, 0.3, 0.1\}$ , respectively. As before, the larger the number of  $L_P$  coefficients is, the smaller the difference between theoretical analysis and simulations is. It is worth noting that the theoretical results do not consider the actual values of  $h_k^{(\text{SP})}$ , as required in (41),<sup>4</sup> since they make use of the asymptotic approximation (43). As can be verified, the analytical results closely match the actual performance of the PRake receivers, especially recalling that the results are not averaged. Only a single random channel realization is in fact considered, because we want to emphasize that not only this approximation is accurate on average, but also that the normalized mean square error (nmse)  $\text{nmse}(u_k^*) = \mathbb{E}\{[(u_{k_A}^*/\Psi - u_k^*)/u_k^*]^2\}$  is considerably low, where  $\mathbb{E}\{\cdot\}$  denotes expectation;  $u_{k_A}^*$  and  $\Psi$  are *computed* following (41) and (43), respectively; and  $u_k^*$  represents the *experimental* utility at the Nash equilibrium. In fact, by averaging over 10 000 channel realizations using the same network parameters,  $\text{nmse}(u_k^*) = \{1.4 \times 10^{-3}, 5.9 \times 10^{-3}, 6.3 \times 10^{-2}\}$  for  $r = \{0.5, 0.3, 0.1\}$ , respectively. As a conclusion, this allows every network fulfilling the above described hypotheses to be studied with the proposed tools.

Fig. 7 shows the loss  $\Psi$  versus the ratio  $r$  for some values of  $\Lambda$  and  $\rho$ . The network parameters are set as follows:  $K = 8$ ,  $N_f = 20$ , and  $L = 200$ . The solid lines represent  $\Lambda = 0 \text{ dB}$ , while the dashed lines depict  $\Lambda = 10 \text{ dB}$ . The circles represent  $N_c = 50$  (and thus  $\rho = 0.25$ ), while the square markers report  $N_c = 200$  (and thus  $\rho = 1.0$ ). As is obvious,  $\Psi$  is a decreasing function of  $r$ . Furthermore,  $\Psi$  is a decreasing function of  $\Lambda$ , since the received power associated to the paths neglected by the PRake receiver is lower as  $\Lambda$  increases. Similarly, keeping the number of multiple paths  $L$  fixed,  $\Psi$  decreases as  $\rho$  increases. This complies with theory [6], [14], since increasing the processing gain provides higher robustness against multipath. As a consequence, a system with a lower  $\rho$  benefits more from a higher number of fingers at the receiver than a system with a higher  $\rho$  does. Hence, when  $\rho$  is lower, a PRake receiver performs worse, i.e.,  $\Psi$  is higher.

It is worth stating that the proposed analysis is mainly focused on energy efficiency. Hence, the main performance index here is represented by the achieved utility at the Nash equilibrium. However, more traditional measures of performance such as SINR or bit error rate (BER) can be obtained using the parameters derived here. In fact, typical target SINRs at the access point can be computed using  $\gamma_k^* = \Gamma(N/\nu(\Lambda, r, \rho))$ , as derived in the previous sections. Similarly, the BER can be approximated by  $Q(\sqrt{\gamma_k^*})$  [14], where  $Q(\cdot)$  denotes the complementary cumulative distribution function of a standard normal random variable.

<sup>4</sup>This is also valid for the case ARake, since  $h_k^{(\text{SP})} = h_k$ .

## V. CONCLUSION

In this paper, we have used a large-system analysis to study performance of PRake receivers using maximal ratio combining when energy-efficient power control techniques are adopted. We have considered a wireless data network in frequency-selective environments, where the user terminals transmit IR-UWB signals to a common concentration point. Assuming the averaged power delay profile and the amplitude of the path coefficients to be exponentially decaying and Rayleigh-distributed, respectively, we have obtained a general characterization for the terms due to multiple access interference and self-interference. The expressions are dependent only on the network parameters and the number of PRake coefficients. A measure of the loss of the PRake receivers with respect to the ARake receiver has then been proposed which is completely independent of the channel realizations. This theoretical approach may also serve as a criterion for network design, since it is completely described by the network parameters.

## APPENDIX

### A. Proof of Prop. 1

To derive (30), we make use of the result (17) of Theorem 1. Using the hypotheses shown in Sect. III,  $\mathbf{D}_k^\alpha$  and  $\mathbf{D}_k^\beta$  are represented by (27) and (28), respectively.

Hence, focusing on the denominator of (17),

$$\varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta) = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{l=1}^L \{\mathbf{D}_k^\alpha \mathbf{D}_k^\beta\}_l = \lim_{L \rightarrow \infty} \frac{\sigma_k^2}{L} \sum_{l=1}^{rL} \Lambda^{-\frac{l-1}{L-1}} = \sigma_k^2 \cdot \frac{\Lambda^r - 1}{\Lambda^r \log \Lambda}. \quad (44)$$

Analogously,

$$\varphi(\mathbf{D}_j^\alpha \mathbf{D}_j^\beta) = \sigma_j^2 \cdot \frac{\Lambda^r - 1}{\Lambda^r \log \Lambda}. \quad (45)$$

Using (7), (8) and (25), after some algebraic manipulation, we obtain

$$\{\mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H}\}_u = \frac{\sigma_j^2}{L} \left( \sum_{m=l+1}^L \Lambda^{-\frac{m-1}{L-1}} \right) u[L-1-l], \quad (46)$$

$$\{\mathbf{C}_k^\beta \mathbf{C}_k^{\beta H}\}_u = \frac{\sigma_k^2}{L} \left( \sum_{m=l+1}^{rL} \Lambda^{-\frac{m-1}{L-1}} \right) u[rL-1-l], \quad (47)$$

where  $u[\cdot]$  is defined as in (29). The terms in the numerator of (17) thus translate into

$$\begin{aligned} \varphi(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha) &= \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{l=1}^L \{\mathbf{D}_j^\alpha\}_l^2 \{\mathbf{C}_k^\beta \mathbf{C}_k^{\beta H}\}_u \\ &= \lim_{L \rightarrow \infty} \frac{\sigma_k^2 \sigma_j^2}{L^2} \sum_{l=1}^{rL-1} \Lambda^{-\frac{l-1}{L-1}} \sum_{m=l+1}^{rL} \Lambda^{-\frac{m-1}{L-1}} = \sigma_k^2 \sigma_j^2 \cdot \frac{\Lambda^{-2r} (\Lambda^r - 1)^2}{2 (\log \Lambda)^2} \end{aligned} \quad (48)$$



and

$$\begin{aligned}\varphi\left(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta\right) &= \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{l=1}^L \left\{ \mathbf{D}_k^\beta \right\}_l^2 \left\{ \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \right\}_{ll} = \lim_{L \rightarrow \infty} \frac{\sigma_k^2 \sigma_j^2}{L^2} \sum_{l=1}^{rL} \Lambda^{-\frac{l-1}{L-1}} \sum_{m=l+1}^L \Lambda^{-\frac{m-1}{L-1}} \\ &= \sigma_k^2 \sigma_j^2 \cdot \frac{\Lambda^{-1-2r} (\Lambda^r - 1) (\Lambda - 2\Lambda^r + \Lambda^{r+1})}{2 (\log \Lambda)^2}.\end{aligned}\quad (49)$$

Using (44)-(45) and (48)-(49),

$$\frac{h_{kj}^{(\text{MAI})}}{h_j^{(\text{SP})}} \xrightarrow{\text{a.s.}} \frac{1}{N} \cdot \frac{\varphi\left(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha\right) + \varphi\left(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta\right)}{\varphi\left(\mathbf{D}_j^\alpha \mathbf{D}_j^\beta\right) \cdot \varphi\left(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta\right)} = \frac{1}{N} \cdot \frac{(\Lambda - 1) \Lambda^{r-1}}{\Lambda^r - 1}.\quad (50)$$

Using (50), the result (30) is straightforward.

### B. Proof of Prop. 2

To derive (32), we make use of the result (18) of Theorem 1. Using the hypotheses shown in Sect. III,  $\mathbf{D}_k^\alpha$  and  $\mathbf{D}_k^\beta$  are represented by (27) and (28), respectively. The denominator can be obtained following the same steps as in App. A:

$$\left( \varphi\left(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta\right) \right)^2 = \sigma_k^4 \cdot \frac{(\Lambda^r - 1)^2}{\Lambda^{2r} (\log \Lambda)^2}.\quad (51)$$

Following (24),

$$\theta_k^2(l, L + l - i) = \sigma_k^4 \cdot \Lambda^{-\frac{L+2l-i-2}{L-1}} \cdot w[l, i],\quad (52)$$

where

$$\begin{aligned}w[l, i] &= u[rL - l] + u[rL - L + i - l] \\ &\quad + 2u[rL - l] \cdot u[rL - L + i - l]\end{aligned}\quad (53)$$

has been introduced for convenience of notation.

In order to obtain explicit expressions for  $w[l, i]$ , it is convenient to split the range of  $r$  into the two following cases.

- $r \leq 1/2$ : taking into account all the possible values of  $l$  and  $i$ ,

$$w[l, i] = \begin{cases} 4, & \text{if } L - rL + 1 \leq i \leq L - 1 \text{ and } 1 \leq l \leq rL - L + i; \\ 1, & \text{either if } 1 \leq i \leq rL \text{ and } 1 \leq l \leq 1, \\ & \text{or if } rL \leq i \leq L - rL \text{ and } 1 \leq l \leq rL, \\ & \text{or if } L - rL + 1 \leq i \leq L - 1 \text{ and } rL - L + i + 1 \leq l \leq rL; \\ 0, & \text{elsewhere.} \end{cases}\quad (54)$$

Substituting (24) and (54) in the numerator of (18) yields

$$\begin{aligned}
& \frac{1}{\sigma_k^4} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L+l-i) = \\
& = \sum_{i=1}^{rL} \phi_i^2 \cdot \sum_{l=1}^i \Lambda^{-\frac{L+2l-i-2}{L-1}} + \sum_{i=rL+1}^{L-rL} \phi_i^2 \cdot \sum_{l=1}^{rL} \Lambda^{-\frac{L+2l-i-2}{L-1}} \\
& + \sum_{i=L-rL+1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^{rL-L+i} 4\Lambda^{-\frac{L+2l-i-2}{L-1}} + \sum_{i=L-rL+1}^{L-1} \phi_i^2 \cdot \sum_{l=rL-L+i+1}^{rL} \Lambda^{-\frac{L+2l-i-2}{L-1}}; \tag{55}
\end{aligned}$$

- $r \geq 1/2$ : taking into account all the possible values of  $l$  and  $i$ ,

$$w[l, i] = \begin{cases} 4, & \text{either if } L - rL + 1 \leq i \leq rL \text{ and } 1 \leq l \leq rL - L + i, \\ & \text{or if } rL + 1 \leq i \leq L - 1 \text{ and } 1 \leq l \leq rL - L + i; \\ 1, & \text{either if } 1 \leq i \leq L - rL \text{ and } 1 \leq l \leq 1, \\ & \text{or if } L - rL + 1 \leq i \leq rL \text{ and } rL - L + i + 1 \leq l \leq i, \\ & \text{or if } rL + 1 \leq i \leq L - 1 \text{ and } rL - L + i + 1 \leq l \leq rL; \\ 0, & \text{elsewhere.} \end{cases} \tag{56}$$

Substituting (24) and (56) in the numerator of (18) yields

$$\begin{aligned}
& \frac{1}{\sigma_k^4} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L+l-i) = \sum_{i=1}^{L-rL} \phi_i^2 \cdot \sum_{l=1}^i \Lambda^{-\frac{L+2l-i-2}{L-1}} \\
& + \sum_{i=L-rL+1}^{rL} \phi_i^2 \cdot \sum_{l=1}^{rL-L+i} 4\Lambda^{-\frac{L+2l-i-2}{L-1}} + \sum_{i=L-rL+1}^{rL} \phi_i^2 \cdot \sum_{l=rL-L+i+1}^i \Lambda^{-\frac{L+2l-i-2}{L-1}} \\
& + \sum_{i=rL+1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^{rL-L+i} 4\Lambda^{-\frac{L+2l-i-2}{L-1}} + \sum_{i=rL+1}^{L-1} \phi_i^2 \cdot \sum_{l=rL-L+i+1}^{rL} \Lambda^{-\frac{L+2l-i-2}{L-1}}. \tag{57}
\end{aligned}$$

In order to obtain (33a)-(33e), the explicit values of  $\phi_i^2$  must be used. From (9)-(10) follows

$$\phi_i^2 = \begin{cases} (L-i)/N_c, & \text{either if } N_c \leq L \text{ and } L - N_c + 1 \leq i \leq L - 1, \\ & \text{or if } N_c \geq L \text{ and } 1 \leq i \leq L - 1; \\ 1, & \text{if } N_c \leq L \text{ and } 1 \leq i \leq L - N_c. \end{cases} \tag{58}$$

As in the case of  $r$ , it is convenient to separate the range of  $\rho = N_c/L$  in the following cases.

- $0 \leq \rho \leq \min(r, 1 - r)$ : substituting (58) in (55) and (57), they both yield

$$\begin{aligned} \frac{1}{\sigma_k^4} \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L + l - i) &= \frac{\Lambda(\Lambda^r - 1)(4\Lambda^{2r} + 3\Lambda^\rho - 1)}{2\Lambda^{\rho+2r+1}\rho(\log \Lambda)^3} \\ &\quad - \frac{2\Lambda^{r+\rho}(\Lambda^r + 3\Lambda - 1)\rho \log \Lambda}{2\Lambda^{\rho+2r+1}\rho(\log \Lambda)^3}. \end{aligned} \quad (59)$$

Making use of (18), (51) and (59), the results (32) and (33a) are straightforward.

- $\min(r, 1 - r) \leq \rho \leq \max(r, 1 - r)$  and  $r \leq 1/2$ : substituting (58) in (55) yields

$$\begin{aligned} \frac{1}{\sigma_k^4} \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L + l - i) &= \frac{\Lambda(\Lambda^{2r} - 1)(4\Lambda^\rho - 1)}{2\Lambda^{\rho+2r+1}\rho(\log \Lambda)^3} \\ &\quad - \frac{2\Lambda^{r+\rho}(3\Lambda^r - \rho + \Lambda^r \rho) \log \Lambda}{2\Lambda^{\rho+2r+1}\rho(\log \Lambda)^3}. \end{aligned} \quad (60)$$

Making use of (18), (51) and (60), the results (32) and (33b) are straightforward.

- $\min(r, 1 - r) \leq \rho \leq \max(r, 1 - r)$  and  $r \geq 1/2$ : substituting (58) in (57) yields

$$\begin{aligned} \frac{1}{\sigma_k^4} \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L + l - i) &= \frac{-4\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho}}{2\Lambda^{2+2r+\rho}\rho(\log \Lambda)^3} \\ &\quad + \frac{3\Lambda^{2+2\rho} - 2\rho^{\rho+r+1}(\Lambda^r \rho + 3\Lambda \rho + r - 1) \log \Lambda}{2\Lambda^{2+2r+\rho}\rho(\log \Lambda)^3}. \end{aligned} \quad (61)$$

Making use of (18), (51) and (61), the results (32) and (33c) are straightforward.

- $\max(r, 1 - r) \leq \rho \leq 1$ : substituting (58) into (55) and (57), they both yield

$$\begin{aligned} \frac{1}{\sigma_k^4} \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L + l - i) &= \frac{-\Lambda^{2+2r} - 4\Lambda^{2+\rho} + \Lambda^{2(r+\rho)} + 4\Lambda^{2+2r+\rho}}{2\Lambda^{2+2r+\rho}\rho(\log \Lambda)^3} \\ &\quad - \frac{2\rho^{\rho+r+1}(\Lambda^r \rho + 3\Lambda \rho + r - 1) \log \Lambda}{2\Lambda^{2+2r+\rho}\rho(\log \Lambda)^3}. \end{aligned} \quad (62)$$

Making use of (18), (51) and (62), the results (32) and (33d) are straightforward.

- $\rho = N_c/L \geq 1$ : substituting (58) into (55) and (57), they both yield

$$\frac{1}{\sigma_k^4} \lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_k^2(l, L + l - i) = \frac{2\Lambda(\Lambda^{2r} - 1) - (\Lambda^r + r + 3\Lambda r - 1)\Lambda^r \log \Lambda}{\Lambda^{2r+1}\rho(\log \Lambda)^3}. \quad (63)$$

Making use of (18), (51) and (63), the results (32) and (33e) are straightforward.

### C. Proof of Prop. 3

At the Nash equilibrium, the transmit power for user  $k$  when using an ARake receiver at the access point,  $p_{k_A}^*$ , can be obtained from (15):

$$p_{k_A}^* = \frac{1}{h_k} \cdot \frac{\sigma^2 \Gamma(\gamma_{0,k_A})}{1 - \Gamma(\gamma_{0,k_A}) \cdot (\gamma_{0,k_A}^{-1} + \zeta_{k_A}^{-1})}, \quad (64)$$

where the subscript  $A$  serves to emphasize that we are considering the case of an ARake, and where we have used the fact that  $h_k^{(\text{SP})}$  is equal to the channel gain  $h_k = \alpha_k^H \cdot \alpha_k = \|\alpha_k\|^2$ . Hence, (43) becomes

$$\Psi = \frac{h_k}{h_k^{(\text{SP})}} \cdot \frac{f(\Gamma(\gamma_{0,k_A}))}{f(\Gamma(\gamma_{0,k}))} \cdot \frac{\Gamma(\gamma_{0,k})}{\Gamma(\gamma_{0,k_A})} \cdot \frac{1 - \Gamma(\gamma_{0,k_A}) \cdot (\gamma_{0,k_A}^{-1} + \zeta_{k_A}^{-1})}{1 - \Gamma(\gamma_{0,k}) \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1})}. \quad (65)$$

To show that  $\Psi$  converges a.s. to the non-random limit of (43), it is convenient to rewrite the ratio  $h_k/h_k^{(\text{SP})}$  as

$$\frac{h_k}{h_k^{(\text{SP})}} = \frac{\frac{1}{L} \alpha_k^H \cdot \alpha_k}{\frac{1}{L} \beta_k^H \cdot \alpha_k}. \quad (66)$$

It is possible to prove [6] that

$$\frac{1}{L} \alpha_k^H \cdot \alpha_k \xrightarrow{\text{a.s.}} \varphi((D_k^\alpha)^2) \quad (67)$$

and, analogously,

$$\frac{1}{L} \beta_k^H \cdot \alpha_k \xrightarrow{\text{a.s.}} \varphi(D_k^\alpha D_k^\beta). \quad (68)$$

Taking into account (27),

$$\begin{aligned} \varphi((D_k^\alpha)^2) &= \lim_{L \rightarrow \infty} \frac{\sigma_k^2}{L} \sum_{l=1}^L \Lambda^{-\frac{l-1}{L-1}} \\ &= \sigma_k^2 \cdot \frac{\Lambda - 1}{\Lambda \log \Lambda}. \end{aligned} \quad (69)$$

Using (44), (66) and (69),

$$\frac{h_k}{h_k^{(\text{SP})}} \xrightarrow{\text{a.s.}} \mu(\Lambda, r), \quad (70)$$

where  $\mu(\Lambda, r)$  is defined as in (31).

Making use of (30), (32), (36), (37) and (70), when the hypotheses of Theorem 1 hold, (65) converges a.s. to (43).

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TABLE I  
LIST OF PARAMETERS USED IN THE SIMULATIONS.

$M$ , total number of bits per packet	100 b
$D$ , number of information bits per packet	100 b
$R$ , bit rate	100 kb/s
$\sigma^2$ , AWGN power at the receiver	$5 \times 10^{-16}$ W
$\bar{p}$ , maximum power constraint	$1 \mu\text{W}$

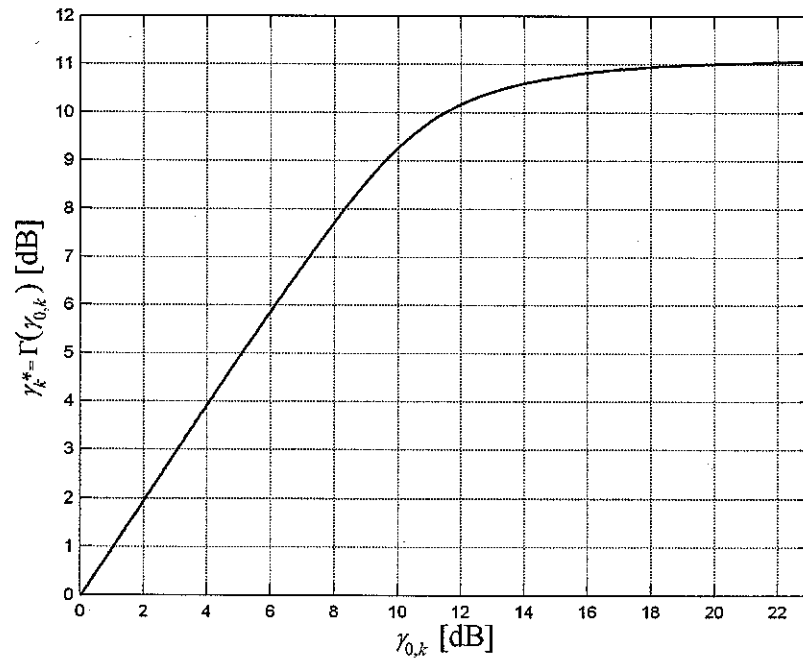


Fig. 1. Shape of  $\gamma_k^*$  as a function of  $\gamma_{0,k}$  ( $M = 100$ ).

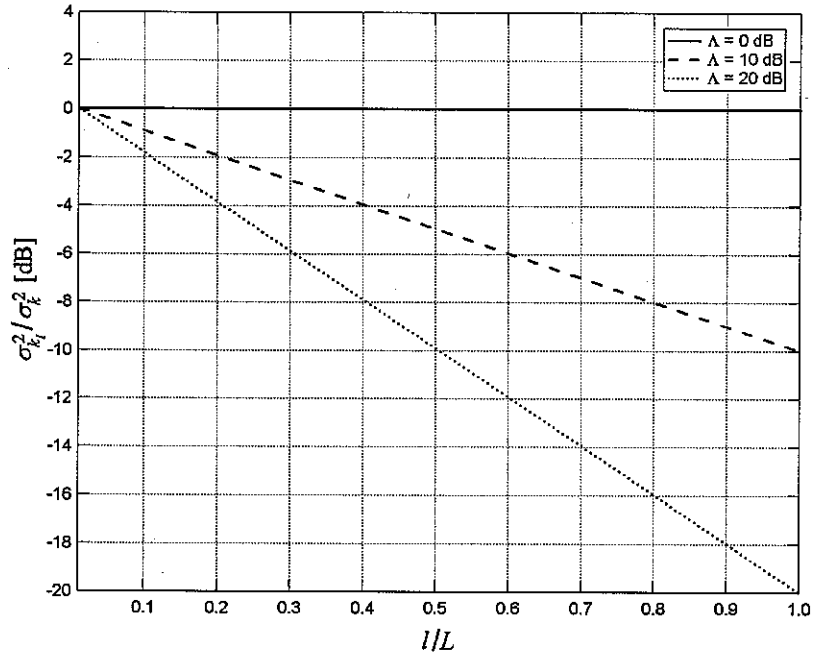


Fig. 2. Average power delay profile versus normalized excess delay.

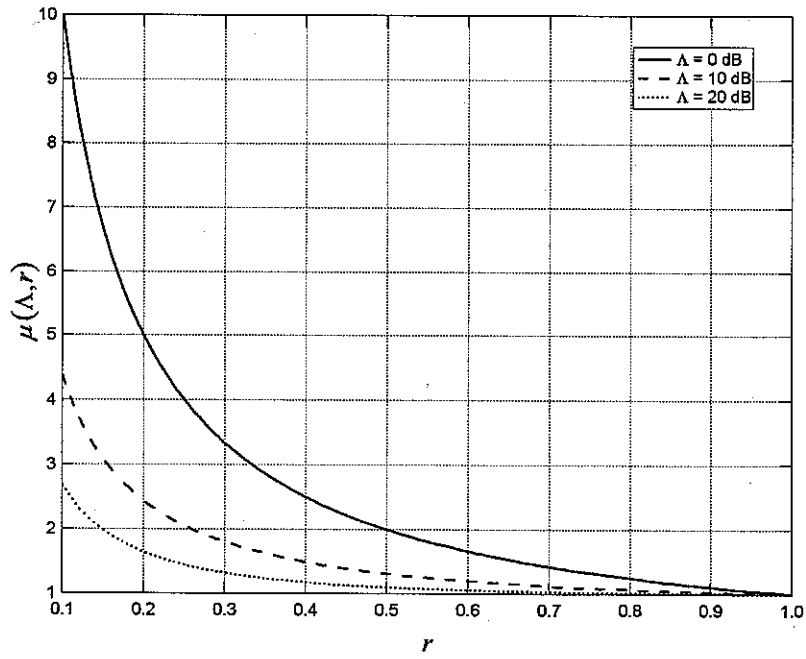


Fig. 3. Shape of  $\mu(\Lambda, r)$  versus  $r$  for some values of  $\Lambda$ .

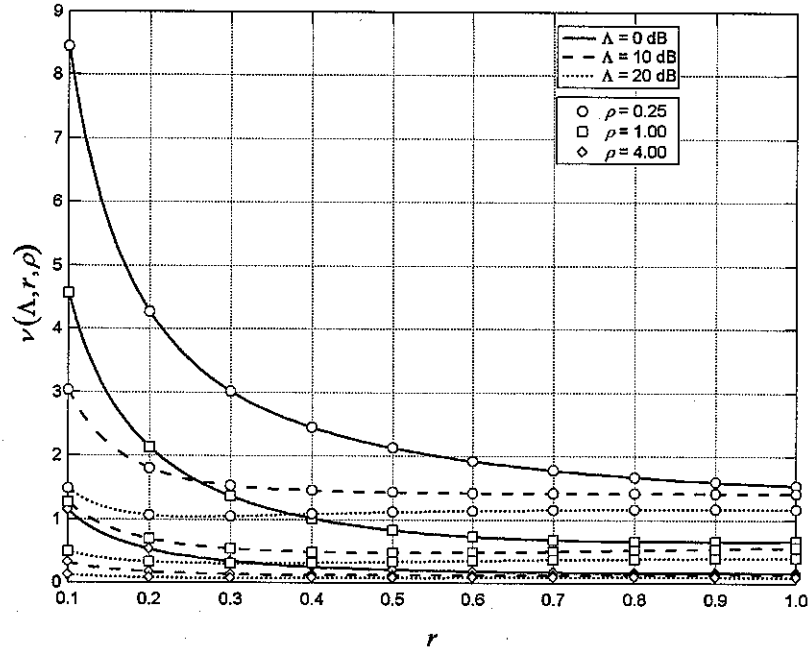


Fig. 4. Shape of  $\nu(\Lambda, r, \rho)$  versus  $r$  for some values of  $\Lambda$  and  $\rho$ .

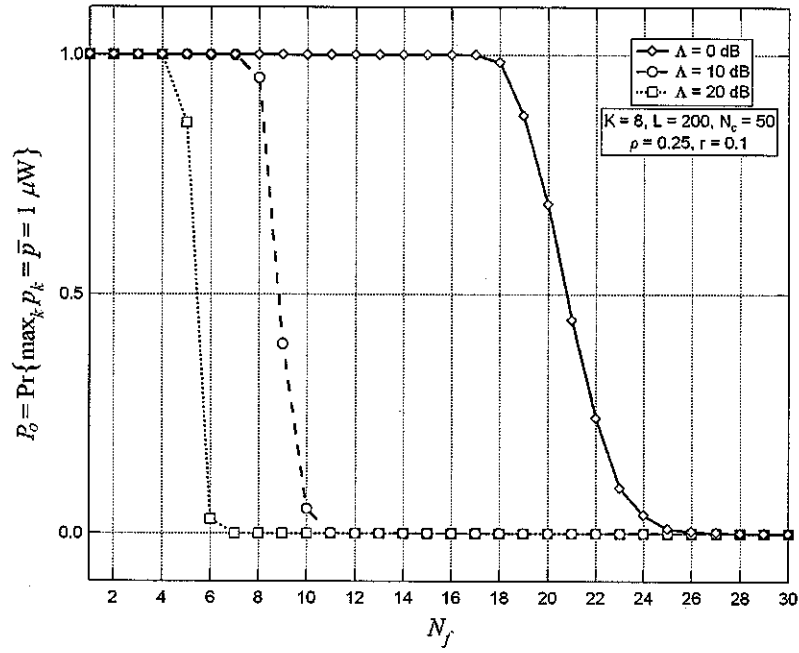


Fig. 5. Probability of having at least one user transmitting at maximum power versus number of frames.



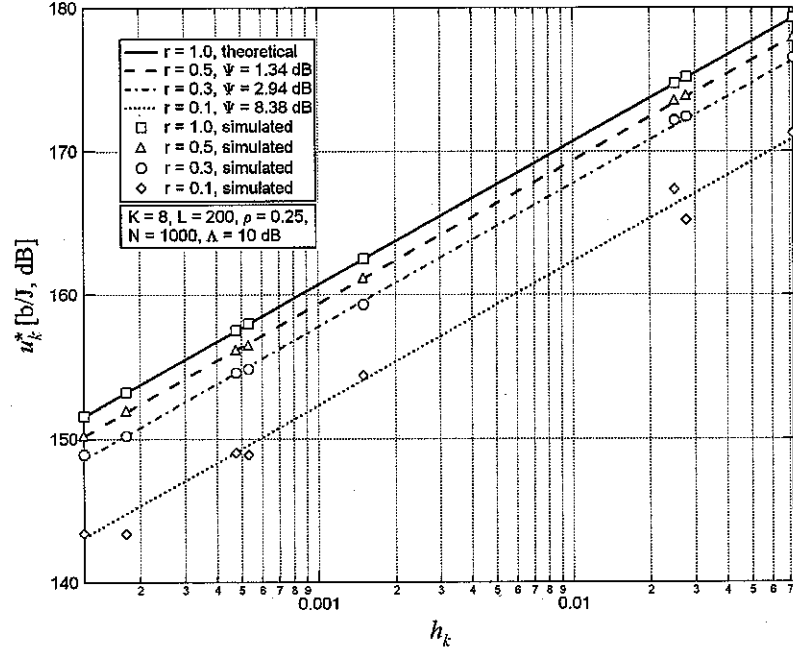


Fig. 6. Achieved utility versus channel gain at the Nash equilibrium for different ratios  $r$ .

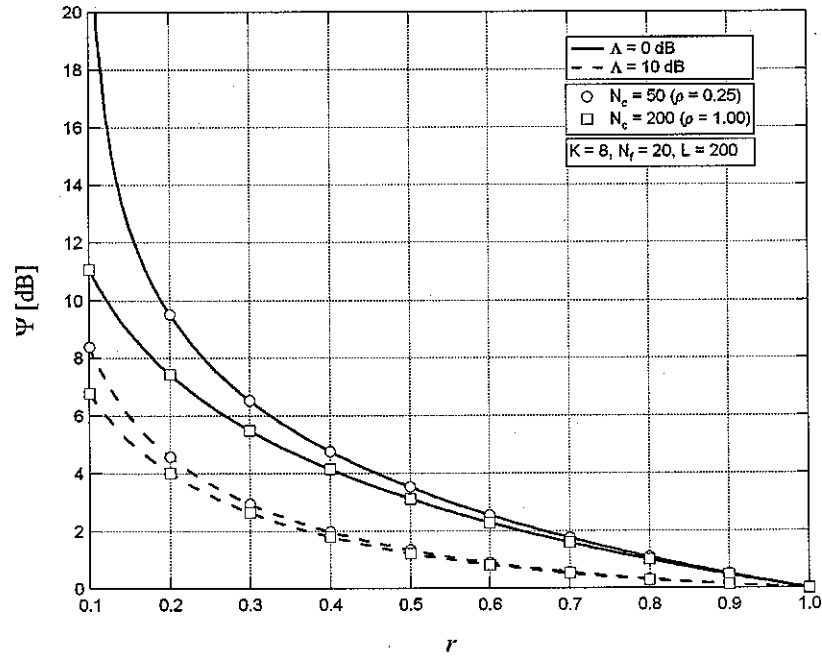


Fig. 7. Shape of the loss  $\Psi$  versus the ratio  $r$  for some values of  $\Lambda$  and  $\rho$ .

# Power Control and Receiver Design for Energy Efficiency in Multipath CDMA Channels with Bandlimited Waveforms

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**Abstract**—This paper is focused on the cross-layer design problem of joint multiuser detection and power control for energy-efficiency optimization in a wireless data network through a game-theoretic approach. Building on work of Meshkati, et al., wherein the tools of game-theory are used in order to achieve energy-efficiency in a simple synchronous code division multiple access system, system asynchronism, the use of bandlimited chip-pulses, and the multipath distortion induced by the wireless channel are explicitly incorporated into the analysis. Several non-cooperative games are proposed wherein users may vary their transmit power and their uplink receiver in order to maximize their utility, which is defined here as the ratio of data throughput to transmit power. In particular, the case in which a linear multiuser detector is adopted at the receiver is considered first, and then, the more challenging case in which non-linear decision feedback multiuser detectors are employed is considered. The proposed games are shown to admit a unique Nash equilibrium point, while simulation results show the effectiveness of the proposed solutions, as well as that the use of a decision-feedback multiuser receiver brings remarkable performance improvements. **Index Terms**—Power control, non-cooperative games, energy-efficiency, CDMA, Multipath fading.

## I. INTRODUCTION

Game theory [1] is a branch of mathematics that has been applied primarily to social science and economics to study the interactions among several autonomous subjects with contrasting interests. Recently, it has been discovered that it can also be used for the design and analysis of communication systems, mostly with application to resource allocation algorithms [2], and, in particular, to power control [3]. As examples, the reader is referred to [4], [5]. Here, for a multiple access wireless data network, noncooperative and cooperative games are introduced, wherein users choose their transmit powers in order to maximize their own utilities, defined as the ratio of the throughput to transmit power. While the above studies consider the issue of power control assuming that a conventional matched filter is available at the receiver, the recent paper [6] considers for the first time the problem

of joint linear receiver design and power control so as to maximize the utility of each user. In particular, it is shown here that the inclusion of receiver design in the considered game brings remarkable advantages, and, also, results based on the powerful large-system analysis are presented.

All of the cited studies, while laying the foundations of the game-theoretic approach to utility maximization in wireless data networks, focus on a very simple model, i.e. a synchronous direct sequence code division multiple access (DS/CDMA) channel subject to flat-fading. In this paper, instead, we extend the game-theoretic framework to a more practical and challenging scenario, namely we explicitly take into account (a) the possible system asynchrony across users; (b) the use of bandlimited chip-pulses; and (c) the multipath distortion induced by the wireless propagation channel. Note that in such a scenario intersymbol and interchip interference arises, thus implying that the appealing mathematical relationships between the signal-to-interference plus noise ratio (SINR) and the transmit power (as revealed in [6]) do not hold any longer, and this makes system analysis much more involved than it is for the case in which no self-interference exists. A further contribution of this paper is the consideration of non-linear multiuser receivers. Indeed, while previous studies have considered the case in which either a matched filter (see, e.g., [5]) or a linear multiuser detector [6] is adopted at the uplink receiver, here we also consider the case in which a non-linear decision feedback receiver is employed at the receiver.

**Notation:**  $(\cdot)^T$  denotes transpose, while  $*$  and  $\times$  denote linear convolution and ordinary product, respectively.

## II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the uplink of an asynchronous DS/CDMA system with  $K$  users, employing bandlimited chip pulses and operating over a frequency-selective fading channel. The received signal at the access point (AP) may be written as<sup>1</sup>

$$r(t) = \sum_{p=0}^{B-1} \sum_{k=1}^K \sqrt{p_k} b_k(p) s'_k(t - \tau_k - pT_b) * c_k(t) + w(t). \quad (1)$$

<sup>1</sup>For the sake of simplicity we assume here a real signal model; however, the extension to complex signals to account for I and Q components is trivial.

The research was supported in part by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252 and by the U. S. Defense Advanced Research Projects Agency under Grant HR0011-06-1-0052.

In the above expression,  $B$  is the transmitted frame or packet length,  $T_b$  is the bit-interval duration,  $p_k$  and  $\tau_k \geq 0$  denote the transmit power and timing offset of the  $k$ -th user,  $b_k(p) \in \{+1, -1\}$  is the  $k$ -th user's information symbol in the  $p$ -th signaling interval (extension to modulations with a larger cardinality is straightforward). Moreover,  $c_k(t)$  is the impulse response modeling the channel effects between the receiver and the  $k$ -th user's transmitter, while  $w(t)$  is the additive noise term, which is assumed to be a zero-mean, Wide-Sense Stationary (WSS) white Gaussian process with Power Spectral Density (PSD)  $N_0/2$ . It is also assumed that the channel coherence time exceeds the packet duration  $BT_b$ , so that the channel impulse responses  $c_0(t), \dots, c_{K-1}(t)$  may be assumed to be time-invariant over each transmitted frame. As to  $s'_k(t)$ , it is the  $k$ -th user's signature waveform and is written as  $s'_k(t) = \sum_{n=0}^{N-1} \beta_k^{(n)} h_{\text{SRRC}}(t - nT_c)$ , with  $\{\beta_k^{(n)}\}_{n=0}^{N-1}$  the  $k$ -th user's spreading sequence,  $N$  the processing gain,  $T_c = T_b/N$  the chip interval, and  $h_{\text{SRRC}}(\cdot)$  a square-root raised cosine waveform with roll-off factor  $\alpha \in [0, 1]$ . We assume here that  $h_{\text{SRRC}}(t)$  is zero outside the interval  $[0, 4T_c]$  and attains its maximum value in  $t = 2T_c$ .

The receiver front-end consists of a filter with impulse response  $h_{\text{SRRC}}(-t)$ , followed by a sampler at rate  $M/T_c$ ; in our simulations we will assume that  $M = 2$ . Denoting by  $y(t)$  the signal at the output of the receiver matched filter, it can be easily shown that

$$y(t) = r(t) * h_{\text{SRRC}}(-t) = \sum_{p=0}^{B-1} \sum_{k=1}^K \sqrt{p_k} b_k(p) \times s_k(t - \tau_k - pT_b) * c_k(t) + \underbrace{w(t) * h_{\text{SRRC}}(-t)}_{n(t)}, \quad (2)$$

$$\text{with } s_k(t) = s'_k(t) * h_{\text{SRRC}}(-t) = \sum_{n=0}^{N-1} \beta_k^{(n)} h_{\text{RC}}(t - nT_c), \text{ and } h_{\text{RC}}(t) = h_{\text{SRRC}}(t) * h_{\text{SRRC}}(-t).$$

Denoting by  $h_k(t) = s_k(t - \tau_k) * c_k(t)$  the *effective* signature waveform for the  $k$ -th user in the  $p$ -th signaling interval, the signal (2) can be expressed as

$$y(t) = \sum_{p=0}^{B-1} \sum_{k=1}^K \sqrt{p_k} b_k(p) h_k(t - pT_b) + n(t). \quad (3)$$

Notice that the waveform  $h_k(t)$  is supported in the interval  $[\tau_k, \tau_k + T_b + T_m + 7T_c]$ , where  $T_m$  denotes the maximum channel multipath delay spread over the  $K$  active users. Assuming that  $\tau_k + T_m < T_b$ , the support of the waveform  $h_k(t - pT_b)$  is contained in the interval  $[pT_b, (p+2)T_b + 7T_c]$ , thus implying that, for a system with processing gain larger than 7, in the symbol interval  $\mathcal{I}_p = [pT_b, (p+2)T_b]$  the contribution from at most four symbols for each user (i.e. the  $p$ -th, the  $(p-1)$ -th, the  $(p-2)$ -th and the  $(p+1)$ -th ones) is observed. Accordingly, sampling the waveform  $y(t)$  at rate  $M/T_c$ , the  $2MN$ -dimensional vector  $y(p)$  collecting the data

samples of the interval  $\mathcal{I}_p$  can be expressed as

$$y(p) = \sum_{k=1}^K \sqrt{p_k} [b_k(p-2)h_{k,-2} + b_k(p-1)h_{k,-1} + b_k(p)h_{k,0} + b_k(p+1)h_{k,+1}] + n(p). \quad (4)$$

In (4), the vector  $h_{k,i}$  is  $2MN$ -dimensional, and contains the samples of the signature  $h_k(t - (p+i)T_b)$  coming from  $\mathcal{I}_p$ , while the vector  $n(p)$  contains the noise contribution, and is a Gaussian random vector with covariance matrix  $M$ . We assume that the data vector  $y(p)$  will be used in order to detect the information symbols  $b_1(p), b_2(p), \dots, b_K(p)$ , i.e. the  $p$ -th epoch data symbols for all the users.

Assume now that each mobile terminal is interested both in having its data received with as small as possible error probability at the AP, and in making optimal use of the energy stored in its battery. Obviously, these are conflicting goals, since error-free reception may be achieved by increasing the received SNR, i.e. by increasing the transmit power, which of course comes at the expense of battery life<sup>2</sup>. A useful approach to quantify these conflicting goals is to define the utility of the  $k$ -th user as the ratio of its throughput, defined as the number of information bits that are received with no error in the unit time, to its transmit power [4], [5], i.e.

$$u_k = \frac{T_k}{p_k}. \quad (5)$$

Note that  $u_k$  is measured in bits/Joule. Denoting by  $R$  the common rate of the network and assuming that each packet of  $B$  bits contains  $L$  information bits and  $B-L$  overhead bits, reserved, e.g., for channel estimation and/or parity checks, the throughput  $T_k$  can be expressed as

$$T_k = R \frac{L}{B} P_k \quad (6)$$

wherein  $P_k$  denotes the probability that a packet from the  $k$ -th user is received error-free. In the considered DS/CDMA setting, the term  $P_k$  depends formally on a number of parameters such as the spreading codes of all the users, their transmit powers and their channel impulse responses; however, a customary approach is to model the overall interference as a Gaussian random process, and assume that  $P_k$  is an increasing function of the  $k$ -th user's SINR  $\gamma_k$ , which is a good model for many practical scenarios.

For the case in which a linear receiver is used to detect the data symbol  $b_k(p)$ , according, i.e., to the decision rule

$$\hat{b}_k(p) = \text{sign} \left[ d_k^T y(p) \right], \quad (7)$$

with  $\hat{b}_k(p)$  the estimate of  $b_k(p)$  and  $d_k$  the  $2NM$ -dimensional vector representing the receive filter for user  $k$ , it is easily seen

<sup>2</sup>Of course there are many other strategies to lower the data error probability, such as for example the use of error correcting codes, diversity exploitation, and implementation of optimal reception techniques at the receiver. Here, however, we are mainly interested in energy efficient data transmission and power usage, so we assume that only the transmit power and the receiver strategy can be varied to achieve energy efficiency.

that for the case at hand the SINR  $\gamma_k$  can be written as

$$\gamma_k = \frac{p_k(d_k^T h_{k,0})^2}{d_k^T M d_k + \sum_{i \neq k} \sum_{j=-2}^1 p_i(d_k^T h_{i,j})^2 + \sum_{j \neq 0} p_k(d_k^T h_{k,j})^2} \quad (8)$$

The exact shape of  $P_k(\gamma_k)$  depends not only on  $\gamma_k$ , but also on other factors such as the modulation and coding type. However, in all cases of relevant interest, it is an increasing function of  $\gamma_k$  with a sigmoidal shape, and converges to unity as  $\gamma_k \rightarrow +\infty$ ; as an example, for binary phase-shift-keying (BPSK) modulation coupled with no channel coding, it is easily shown that

$$P_k(\gamma_k) = \left[1 - Q(\sqrt{2\gamma_k})\right]^B, \quad (9)$$

with  $Q(\cdot)$  the complementary cumulative distribution function of a zero-mean random Gaussian variate with unit variance.

It should be noted however that substituting Eq. (9) into (6), and, in turn, into (5), leads to a strong incongruence. Indeed, for  $p_k \rightarrow 0$ , we have  $\gamma_k \rightarrow 0$ , but  $P_k$  converges to a small but non-zero value (i.e.  $2^{-B}$ ), thus implying that an unboundedly large utility can be achieved by transmitting with zero power. To circumvent this problem, a customary approach [5], [6] is to replace  $P_k$  with an efficiency function, say  $f_k(\gamma_k)$ , whose behavior should approximate as close as possible that of  $P_k$ , except that for  $\gamma_k \rightarrow 0$  it is required that  $f_k(\gamma_k) = o(\gamma_k)$ . The function  $f(\gamma_k) = (1 - e^{-\gamma_k/2})^B$  is a widely accepted substitute for the true probability of correct packet reception, and in the following we will adopt this model. This efficiency function is increasing and S-shaped, converges to unity as  $\gamma_k$  approaches infinity, and has a continuous first order derivative<sup>3</sup>.

Summing up, substituting (6) into (5) and replacing the probability  $P_k$  with the above defined efficiency function, we obtain the following expression for the  $k$ -th user's utility:

$$u_k = R \frac{L}{B} \frac{f(\gamma_k)}{p_k}, \quad \forall k = 1, \dots, K. \quad (10)$$

### III. NON-COOPERATIVE GAMES WITH LINEAR RECEIVERS

In what follows we illustrate three different noncooperative games wherein each user aims at maximizing its own utility by varying its transmit power, and, possibly, its linear uplink receiver. Formally, the considered game  $\mathcal{G}$  can be described as the triplet  $\mathcal{G} = [\mathcal{K}, \{S_k\}, \{u_k\}]$ , wherein  $\mathcal{K} = \{1, 2, \dots, K\}$  is the set of active users participating in the game,  $u_k$  is the  $k$ -th user's utility defined in eq. (10), and

$$S_k = [0, P_{k,\max}] \times \mathcal{R}^{2NM}, \quad (11)$$

is the set of possible actions (strategies) that user  $k$  can take. It is seen that  $S_k$  is written as the Cartesian product of two different sets, and indeed  $[0, P_{k,\max}]$  is the range of available transmit powers for the  $k$ -th user (note that  $P_{k,\max}$  is the maximum allowed transmit power of user  $k$ ), while  $\mathcal{R}^{2NM}$ , with  $\mathcal{R}$  the real line, defines the set of all possible linear receive filters.

<sup>3</sup>Note that we have omitted the subscript  $k$ , i.e. we have used the notation  $f(\gamma_k)$  in place of  $f_k(\gamma_k)$  since we assume that the efficiency function is the same for all the users.

#### A. Power control with plain matched filter

We first consider the case in which  $S_k = [0, P_{k,\max}]$  and the uplink receiver is a matched filter, i.e. we assume that each user tunes its transmit power in order to maximize its own utility, but the uplink receiver is a matched filter<sup>4</sup>. Consequently, the  $k$ -th user's SINR is expressed as

$$\gamma_k = \frac{p_k \|h_{k,0}\|^4}{h_{k,0}^T M h_{k,0} + \sum_{i \neq k} \sum_{j=-2}^1 p_i (h_{k,0}^T h_{i,j})^2 + \sum_{j \neq 0} p_k (h_{k,0}^T h_{k,j})^2} \quad (12)$$

and the noncooperative game can be cast as the following maximization problem

$$\max_{S_k} u_k = \max_{p_k \in [0, P_{k,\max}]} u_k(p_k) = \max_{p_k \in [0, P_{k,\max}]} \frac{f(\gamma_k(p_k))}{p_k}, \quad (13)$$

$\forall k = 1, \dots, K$ . Now, the following result can be stated about maximization (13).

**Proposition 1:** *The non-cooperative game defined in (13) admits a unique Nash equilibrium point  $p_k^*$ , for  $k = 1, \dots, K$ , wherein  $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  denoting the  $k$ -th user's transmit power such that the  $k$ -th user's SINR  $\gamma_k$  equals  $\bar{\gamma}_k$ , i.e. the unique solution of the equation*

$$\frac{B}{2a_k} \gamma(a_k - b_k \gamma) = \exp(\gamma/2) - 1, \quad (14)$$

with  $a_k = \|h_{k,0}\|^4$  and

$$b_k = \sum_{j \neq 0} (h_{k,0}^T h_{k,j})^2.$$

**Proof:** The proof is omitted for brevity. ■

In summary, Proposition 1 states that a Nash equilibrium for the noncooperative game (13) always exists, and it can be found with the following steps. First, the unique solution  $\bar{\gamma}_k$  of the equation (14) is determined. Then, each user adjusts its transmit power to achieve its target SINR  $\bar{\gamma}_k$ . These steps are repeated until convergence is reached.

#### B. Power control and receiver design with no ISI

Let us now consider the case in which not only the transmit power, but also the linear receiver can be tuned so as to maximize utility for each user; moreover, let us also impose the condition that the receive filter be orthogonal to the subspace spanned by ISI. Denoting by  $O_k$  a  $2NM \times (2NM - 3)$ -dimensional matrix containing in its columns a basis for the orthogonal complement of the subspace spanned by the  $k$ -th user's ISI, i.e. by the vectors  $h_{k,-2}$ ,  $h_{k,-1}$ , and  $h_{k,1}$ , we assume that the decision rule to detect the symbol  $b_k(p)$  can be written as

$$\hat{b}_k(p) = \text{sign} \left[ x_k^T O_k^T y(p) \right], \quad (15)$$

<sup>4</sup>Note that for an oversampling factor  $M > 1$  a whitening transformation would in principle be required prior to matched filtering; for the sake of simplicity, however, noise whitening is not performed here.

with  $\mathbf{x}_k$  a  $(2NM - 3)$ -dimensional vector. The  $k$ -th user's SINR is now written as

$$\gamma_k = \frac{p_k (\mathbf{x}_k^T \mathbf{O}_k^T \mathbf{h}_{k,0})^2}{\mathbf{x}_k^T \mathbf{O}_k^T \mathbf{M} \mathbf{O}_k \mathbf{x}_k + \sum_{i \neq k} \sum_{j=-2}^1 p_i (\mathbf{x}_k^T \mathbf{O}_k^T \mathbf{h}_{i,j})^2}, \quad (16)$$

namely the  $k$ -th user's transmit power appears only in the numerator in the RHS of (16), thus implying that the relation  $\frac{d\gamma_k}{dp_k} = \frac{\gamma_k}{p_k}$  holds. We now consider the following maximization problem

$$\max_{S_k} u_k = \max_{p_k, \mathbf{x}_k} u_k(p_k, \mathbf{x}_k), \quad \forall k = 1, \dots, K. \quad (17)$$

Given (10), the above maximization can be also written as

$$\max_{p_k, \mathbf{x}_k} \frac{f(\gamma_k(p_k, \mathbf{x}_k))}{p_k} = \max_{p_k} \frac{f\left(\max_{\mathbf{x}_k} \gamma_k(p_k, \mathbf{x}_k)\right)}{p_k}, \quad (18)$$

i.e. we can first take care of SINR maximization with respect to linear receivers, and then focus on maximization of the resulting utility with respect to transmit power. We now have the following:

**Proposition 2:** Let  $\mathbf{M}_{yy}$  denote the covariance matrix of the vector  $\mathbf{y}(p)$ . The non-cooperative game defined in (17) admits a unique Nash equilibrium point  $(p_k^*, \mathbf{x}_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $\mathbf{x}_k^* = \sqrt{p_k^*} (\mathbf{O}_k^T \mathbf{M}_{yy} \mathbf{O}_k)^{-1} \mathbf{O}_k^T \mathbf{h}_{k,0}$  is the unique (up to a positive scaling factor)  $k$ -th user's receive filter that maximizes the SINR  $\gamma_k$  in (16). Denote  $\gamma_k^* = \max_{\mathbf{x}_k} \gamma_k$ .
- $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  the  $k$ -th user's transmit power such that the  $k$ -th user's maximum SINR  $\gamma_k^*$  equals  $\bar{\gamma}$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  denoting the derivative of  $f(\gamma)$ .

**Proof:** The proof is omitted due to lack of space. Note however that, due to the constraint that the receive filter is orthogonal to the ISI contribution, the mathematical structure of the maximization (17) is similar to that of the noncooperative game proposed in [6], and the proof can thus be adapted from there. ■

The above equilibrium can be reached according to the following procedure. For a given set of users' transmit powers, the receiver filter coefficients can be set according to the relation  $\mathbf{x}_k^* = \sqrt{p_k^*} (\mathbf{O}_k^T \mathbf{M}_{yy} \mathbf{O}_k)^{-1} \mathbf{O}_k^T \mathbf{h}_{k,0}$ ; each user can then tune its power so as to achieve the target SINR  $\bar{\gamma}$ . These steps are repeated until convergence is reached.

### C. Power control and unconstrained receiver design

Finally, we consider the case in which no constraint is imposed on the receive filter, so that the  $k$ -th user's SINR is written as in Eq. (8). We now consider the following maximization

$$\max_{S_k} u_k = \max_{p_k, d_k} u_k(p_k, d_k) = \max_{p_k} \frac{f\left(\max_{d_k} \gamma_k(p_k, d_k)\right)}{p_k}, \quad (19)$$

$\forall k = 1, \dots, K$ , wherein the fact that the efficiency function is non-decreasing has been exploited. Now, the maximization of  $\gamma_k$  with respect to  $d_k$  is trivial, since it is well known that the linear receiver that maximizes SINR is the minimum mean square error multiuser receiver. As a consequence, denoting by  $\bar{d}_k$  the maximizer of  $\gamma_k$ , we have

$$\bar{d}_k = \sqrt{p_k} \mathbf{M}_{yy}^{-1} \mathbf{h}_{k,0}; \quad (20)$$

let us denote by  $\bar{\gamma}_k(p_k)$  the  $k$ -th user's SINR with  $d_k = \bar{d}_k$ . Maximizing the utility with respect to the transmit power requires instead solving the equation

$$f(\bar{\gamma}_k(p_k)) = f'(\bar{\gamma}_k(p_k)) \bar{\gamma}_k(p_k) p_k, \quad (21)$$

with  $(\cdot)'$  denoting first-order derivative with respect to  $p_k$ . Now, (21) appears to be quite complicated and unmanageable. Indeed, note that letting  $\mathbf{H}_k = [\mathbf{h}_{k,-2} \ \mathbf{h}_{k,-1} \ \mathbf{h}_{k,1}]$ , we have

$$\mathbf{M}_{yy} = \mathbf{M}_{yy}(p_k) = \mathbf{Q}_k + p_k \mathbf{H}_k \mathbf{H}_k^T + p_k \mathbf{h}_{k,0} \mathbf{h}_{k,0}^T, \quad (22)$$

with  $\mathbf{Q}_k$  the covariance matrix of the thermal noise and of the multiuser interference for the  $k$ -th user, thus implying that  $\bar{\gamma}_k(p_k)$  is expressed as

$$\bar{\gamma}_k(p_k) = \frac{p_k (\mathbf{h}_{k,0}^T \mathbf{M}_{yy}^{-1}(p_k) \mathbf{h}_{k,0})^2}{\mathbf{h}_{k,0}^T \mathbf{M}_{yy}^{-1}(p_k) \mathbf{h}_{k,0} - p_k (\mathbf{h}_{k,0}^T \mathbf{M}_{yy}^{-1}(p_k) \mathbf{h}_{k,0})^2}. \quad (23)$$

It is clear that substituting (23) and its first-order derivative into (21) and solving with respect to  $p_k$  is quite complicated. Accordingly, we have not been able in this case to formally prove the existence of a Nash equilibrium point. However, we have numerically evaluated the utility function and (21), and we have found in every case considered that (21) admits a unique solution and that the resulting game admits an equilibrium point. We thus state the following conjecture.

**Conjecture 1:** The non-cooperative game defined in (19) admits a unique Nash equilibrium point  $(p_k^*, d_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $d_k^*$  is the linear MMSE receiver (see Eq. (20)), which maximizes the SINR  $\gamma_k$  in (8). Denote  $\bar{\gamma}_k = \max_{d_k} \gamma_k$ .
- $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  the unique solution of Eq. (21).

Also in this case, the equilibrium can be reached through an iterative procedure. For a given set of users' transmit powers, the receiver filter coefficients can be set equal to the MMSE multiuser receiver; each user can then tune its power to  $p_k^*$ , and these steps are repeated until convergence is reached.

## IV. NON-COOPERATIVE GAMES WITH DECISION-FEEDBACK RECEIVERS

Consider now the case in which a non-linear decision feedback receiver is used at the receiver. We assume that the users are indexed according to a non-increasing sorting of their channel gains, i.e. we assume that  $\|\mathbf{h}_{1,0}\| > \|\mathbf{h}_{2,0}\| > \dots, \|\mathbf{h}_{K,0}\|$ . We consider a serial interference cancellation

(SIC) receiver wherein detection of the symbol from the  $k$ -th user is made according to the following rule

$$\hat{b}_k(p) = \text{sign} \left[ d_k^T \left( y(p) - \sum_{j < k} \sum_{i=-2}^0 \sqrt{p_j} \hat{b}_j(p+i) h_{j,i} \right) \right] \quad (24)$$

Accordingly, if past decisions are correct, users that are detected later enjoy a considerable reduction of multiple access interference, and indeed the SINR for user  $k$ , under the assumption of correctness of past decisions, is written as

$$\gamma_k = \frac{p_k (d_k^T h_{k,0})^2}{\zeta_k}, \quad (25)$$

$$\text{with } \zeta_k = d_k^T M d_k + \sum_{j < k} p_j (d_k^T h_{j,1})^2 + \sum_{j \neq 0} p_k (d_k^T h_{k,j})^2 + \sum_{j > k} \sum_{i=-2}^1 p_j (d_k^T h_{j,i})^2.$$

#### A. Power control and receiver design with no ISI

Replicating the path of the previous section, we start imposing the constraint that the receive filter be orthogonal to the ISI subspace for each user, i.e. our decision rule is

$$\hat{b}_k(p) = \text{sign} \left[ x_k^T O_k^T \left( y(p) - \sum_{j < k} \sum_{i=-2}^0 \sqrt{p_j} \hat{b}_j(p+i) h_{j,i} \right) \right] \quad (26)$$

and the  $k$ -th user SINR is

$$\gamma_k = \frac{p_k (x_k^T O_k^T h_{k,0})^2}{\varrho_k}, \quad (27)$$

with  $\varrho_k = x_k^T O_k^T M O_k x_k + \sum_{j < k} p_j (x_k^T O_k^T h_{j,1})^2 + \sum_{j > k} \sum_{i=-2}^1 p_j (x_k^T O_k^T h_{j,i})^2$ . Given receiver (26) and the SINR expression (27), we consider here the problem of utility maximization with respect to the transmit power, and receiver vectors  $x_1, \dots, x_K$ :

$$\max_{p_k, x_k} \frac{f(\gamma_k(p_k, x_k))}{p_k}, \quad \forall k = 1, \dots, K. \quad (28)$$

The following result can be shown to hold.

**Proposition 3:** Let  $J_k$  be a matrix having as columns the vectors in the set

$$\{\sqrt{p_i} h_{i,1}\}_{i=1, \dots, K} \cup \{\sqrt{p_i} h_{i,j}\}_{i \geq k, j=-2, -1, 0}$$

and define  $M_k = (J_k J_k^T + M)$ ; The non-cooperative game defined in (28) admits a unique Nash equilibrium point  $(p_k^*, x_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $x_k^* = \sqrt{p_k} (O_k^T M_k O_k)^{-1} O_k^T h_{k,0}$  is the unique  $k$ -th user receive filter<sup>5</sup> that maximizes the SINR  $\gamma_k$  given in (27). Denote  $\bar{\gamma}_k = \max_{x_k} \gamma_k$ .
- $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  the  $k$ -th user's transmit power such that the  $k$ -th user's maximum SINR  $\bar{\gamma}_k$  equals

$\bar{\gamma}_k$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  the derivative of  $f(\gamma)$ .

**Proof:** The proof is omitted due to lack of space. ■

#### B. Power control and unconstrained receiver design

Finally, we consider the case in which no constraint is imposed on the receive filter, so that the  $k$ -th user's SINR is written as in (25), and the decision rule is given by (24). We now consider the following maximization

$$\max_{S_k} u_k = \max_{p_k, d_k} u_k(p_k, d_k) = \max_{p_k} \frac{f\left(\max_{d_k} \gamma_k(p_k, d_k)\right)}{p_k}, \quad (29)$$

$\forall k = 1, \dots, K$ . Now, denoting by  $\bar{d}_k$  the maximizer of  $\gamma_k$ , it is easy to show that

$$\bar{d}_k = \sqrt{p_k} M_k^{-1} h_{k,0}; \quad (30)$$

let us denote by  $\bar{\gamma}_k(p_k)$  the  $k$ -th user's SINR as  $d_k = \bar{d}_k$ . Maximizing the utility with respect to the transmit power requires instead solving the equation

$$f(\bar{\gamma}_k(p_k)) = f'(\bar{\gamma}_k(p_k)) \bar{\gamma}_k(p_k) p_k. \quad (31)$$

Now, (31) is formally equivalent to (21) and is quite complicated to manage. Accordingly, the same considerations of Section III.C apply here as well, and, supported by extensive computer simulations, we conjecture the existence of a unique Nash equilibrium. We thus have the following

**Conjecture 2:** The non-cooperative game defined in (29) admits a unique Nash equilibrium point  $(p_k^*, d_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $d_k^*$  is given by Eq. (30), which maximizes the user  $k$  SINR  $\gamma_k$  in (25). Denote  $\bar{\gamma}_k = \max_{d_k} \gamma_k$ .
- $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  the unique solution of Eq. (31).

Also in this case, the equilibrium can be reached through an iterative procedure. For a given set of users' transmit powers, the receiver filter coefficients can be set equal to the receiver in (30); each user can then tune its power to  $p_k^*$ , and these steps are repeated until convergence is reached.

## V. NUMERICAL RESULTS

We consider now an uplink DS/CDMA system with processing gain  $N = 7$ , and assume that the packet length is  $B = 120$ . Users may have random positions with a distance from the AP ranging from 10m to 500m. The channel impulse response  $c_k(t)$  for the generic  $k$ -th user is assumed to be equal

$$c_k(t) = \sum_{\ell=1}^3 \alpha_{k,\ell} \delta(t - \tau_{k,\ell}), \quad \forall k = 1, \dots, K \text{ with } \tau_{k,\ell}$$

such that  $\tau_k + \tau_{k,\ell}$  is uniformly distributed in  $[0, T_b]$  and  $\alpha_{k,\ell}$  is a Rayleigh distributed random variate with mean equal to  $d_k^{-2} i_\ell$ , with  $d_k$  being the distance of user  $k$  from the AP, and  $[i_1, i_2, i_3] = [0.5, 0.3, 0.2]$ . For the thermal noise level, we take  $N_0 = 10^{-9}$  W/Hz, while the maximum allowed power  $P_{k,\max}$  is 25dB. We present here results of averaging over

<sup>5</sup>Uniqueness here means up to a positive scaling factor.

5000 independent realizations for the users locations, fading channel coefficients and set of spreading codes.

Figs. 1 - 2 report the achieved average utility (measured in bits/Joule) and the average user transmit power for the proposed non-cooperative games. As expected, the power control game with matched filter at the receiver is the one with the poorest performance, while the best performance is attained by the non-linear decision-feedback receivers. It is seen that for  $K > N$  the average utility achieved by the non-linear receivers is twice the average utility achieved by the linear receivers. Moreover, constrained receivers are outperformed by unconstrained receivers, even though the gap is not that large.

Fig. 3 reports the average fraction of users that transmit at the maximum available power, i.e. the probability that a user implementing a certain game is not able to achieve its target SINR and ends up transmitting at its maximum power. As expected, it is seen that the larger fraction corresponds to the use of a matched filter at the receiver, while using non-linear decision feedback receivers permits minimizing this fraction, which, moreover, increases as the network load (i.e. number of users) increases.

## VI. CONCLUSIONS

In this paper we have considered the problem of utility maximization in a wireless data network through the use of a game-theoretic approach. The cross-layer issue of multiuser receiver design and power control for utility maximization has been considered for the practical scenario of an asynchronous, bandlimited and multipath distorted CDMA system. The case in which a non-linear decision feedback detector is adopted has been considered. First we have derived the non-linear decision feedback receiver maximizing the utility for each user; then, we have shown how the use of a non-linear multiuser receiver provides significant performance gains, especially in the case in which the number of users is close to or larger than the system processing gain. Overall, it can be stated that game theory is an attractive mathematical tool that can be effectively used for the design of utility-maximizing resource allocation algorithms in wireless networks operating in practical scenarios.

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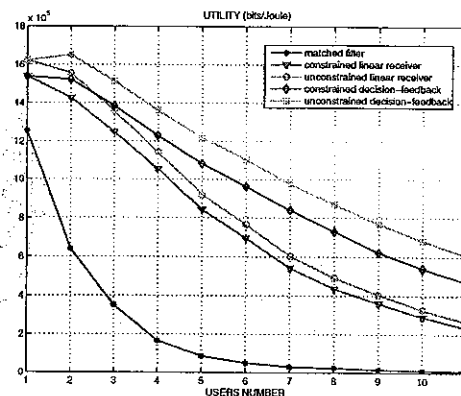


Fig. 1. Achieved average utility versus number of active users for the proposed noncooperative games.

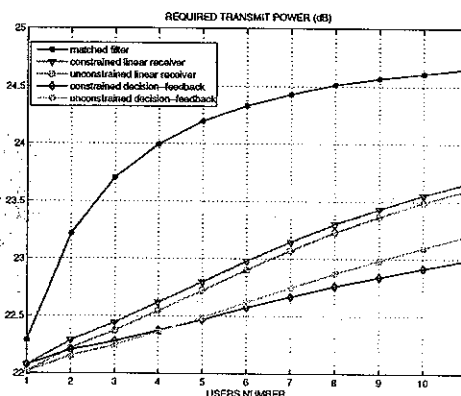


Fig. 2. Average transmit power versus number of active users for the proposed noncooperative games.

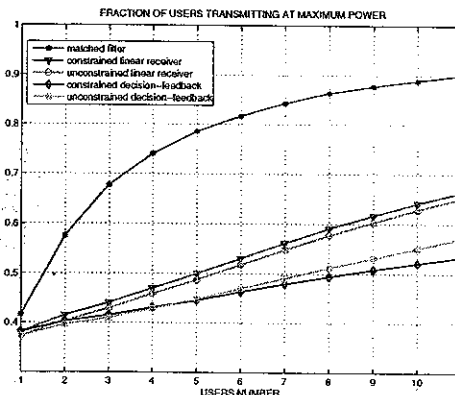


Fig. 3. Average fraction of users transmitting at the maximum power versus number of active users for the proposed noncooperative games.

# Non-cooperative Games for Spreading Code Optimization, Power Control and Receiver Design in Wireless Data Networks

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**Abstract**—This paper focuses on the issue of energy efficiency in wireless data networks through a game theoretic approach. The case considered is that in which each user is allowed to vary its transmit power, spreading code, and uplink receiver in order to maximize its own utility, which is here defined as the ratio of data throughput to transmit power. In particular, the case in which linear multiuser detectors are employed at the receiver is treated first, and, then, the more challenging case in which non-linear decision feedback multiuser receivers are adopted is addressed. It is shown that, for both receivers, the problem at hand of utility maximization can be regarded as a non-cooperative game, and it is proved that a unique Nash equilibrium point exists. Simulation results show that significant performance gains can be obtained through both non-linear processing and spreading code optimization; in particular, for systems with a number of users not larger than the processing gain, remarkable gains come from spreading code optimization, while, for overloaded systems, the largest gains come from the use of non-linear processing. In every case, however, the non-cooperative games proposed here are shown to outperform competing alternatives.

## I. INTRODUCTION

Game theory [1] is a branch of mathematics that has been applied primarily in economics and other social sciences to study the interactions among several autonomous subjects with contrasting interests. More recently, it has been discovered that it can also be used for the design and analysis of communication systems, mostly with application to resource allocation algorithms [2], and, in particular, to power control [3]. As examples, the reader is referred to [4], [5], [6]. Here, for a multiple access wireless data network, noncooperative and cooperative games are introduced, wherein each user chooses its transmit power in order to maximize its own utility, defined as the ratio of the throughput to transmit power. While the above papers consider the issue of power control assuming that a conventional matched filter is available at the receiver, the recent paper [7] considers the problem of joint linear receiver design and power control so as to maximize the utility of each user. It is shown here that the inclusion of receiver design in the considered game brings remarkable advantages, and, also, results based on the powerful large-system analysis are

presented.

This paper is the first in this area that considers the cross-layer issue of utility maximization with respect to the choice of receiver, spreading code and transmit power. First of all, we generalize the game considered in [7] by considering also spreading code optimization. We show that iterative algorithms, of the same kind proposed in [8], can be applied to our scenario in order to improve the achieved Signal-to-Noise plus Interference (SINR) of each user. We will show that the newly considered noncooperative game admits a unique Nash equilibrium and achieves remarkable gains with respect to the performance levels attained by the solution proposed in [7]. Then, we consider the problem of utility maximization with respect to transmit power and spreading code, for the case in which a *non-linear* decision feedback receiver is used. We thus propose two noncooperative games wherein first transmit power is chosen so as to maximize utility, and, then, joint spreading code optimization and power control is undertaken for utility maximization. Our results will show that remarkable gains are granted by the use of spreading code optimization when the number of users does not exceed the processing gain (undersaturated region), while, for saturated systems, non-linear interference cancellation, eventually coupled with code optimization, provides the most significant gains.

The rest of this paper is organized as follows. The next section contains some preliminaries and the system model of interest. Section III introduces a non-cooperative game for the case in which linear receivers are employed, while in Section IV we introduce two non-cooperative utility maximization games for the case that a non-linear interference cancellation receiver is adopted. In Section V we present and discuss the results of some computer simulations that show the merits of the proposed games and their advantages with respect to competing alternatives. Finally, we give some concluding remarks in Section VI.

## II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the uplink of a  $K$ -user synchronous, single-cell, direct-sequence code division multiple access (DS/CDMA)



network with processing gain  $N$  and subject to flat fading. After chip-matched filtering and sampling at the chip-rate, the  $N$ -dimensional received data vector, say  $r$ , corresponding to one symbol interval, can be written as

$$r = \sum_{k=1}^K \sqrt{p_k} h_k b_k s_k + n, \quad (1)$$

wherein  $p_k$  is the transmit power of the  $k$ -th user<sup>1</sup>,  $b_k \in \{-1, 1\}$  is the information symbol of the  $k$ -th user, and  $h_k$  is the real<sup>2</sup> channel gain between the  $k$ -th user's transmitter and the access point (AP); the actual value of  $h_k$  depends on both the distance of the  $k$ -th user's terminal from the AP and the channel fading fluctuations. The  $N$ -dimensional vector  $s_k$  is the spreading code of the  $k$ -th user; we assume that the entries of  $s_k$  are real and that  $s_k^T s_k = \|s_k\|^2 = 1$ , with  $(\cdot)^T$  denoting transpose. Finally,  $n$  is the ambient noise vector, which we assume to be a zero-mean white Gaussian random process with covariance matrix  $(N_0/2)I_N$ , with  $I_N$  the identity matrix of order  $N$ . An alternative and compact representation of (1) is given by

$$r = S P^{1/2} H b + n, \quad (2)$$

wherein  $S = [s_1, \dots, s_K]$  is the  $N \times K$ -dimensional spreading code matrix,  $P$  and  $H$  are  $K \times K$ -dimensional diagonal matrices, whose diagonals are  $[p_1, \dots, p_K]$  and  $[h_1, \dots, h_K]$ , respectively, and, finally,  $b = [b_1, \dots, b_K]^T$  is the  $K$ -dimensional vector of the data symbols.

Assume now that each mobile terminal sends its data in packets of  $M$  bits, and that it is interested both in having its data received with as small as possible error probability at the AP, and in making careful use of the energy stored in its battery. Obviously, these are conflicting goals, since error-free reception may be achieved by increasing the received SNR, i.e. by increasing the transmit power, which of course comes at the expense of battery life<sup>3</sup>. A useful approach to quantify these conflicting goals is to define the utility of the  $k$ -th user as the ratio of its throughput, defined as the number of information bits that are received with no error in unit time, to its transmit power [4], [5], i.e.

$$u_k = \frac{T_k}{p_k}. \quad (3)$$

Note that  $u_k$  is measured in bit/Joule, i.e. it represents the number of successful bit transmissions that can be made for each Joule of energy drained from the battery. Denoting by  $R$  the common rate of the network (extension to the case in which each user transmits with its own rate  $R_k$  is quite simple) and assuming that each packet of  $M$  symbols

contains  $L$  information symbols and  $M - L$  overhead symbols, reserved, e.g., for channel estimation and/or parity checks, the throughput  $T_k$  can be expressed as

$$T_k = R \frac{L}{M} P_k \quad (4)$$

wherein  $P_k$  denotes the probability that a packet from the  $k$ -th user is received error-free. In the considered DS/CDMA setting, the term  $P_k$  depends formally on a number of parameters such as the spreading codes of all the users and the diagonal entries of the matrices  $P$  and  $H$ , as well as on the strength of the used error correcting codes. However, a customary approach is to model the multiple access interference as a Gaussian random process, and assume that  $P_k$  is an increasing function of the  $k$ -th user's Signal-to-Interference plus Noise-Ratio (SINR)  $\gamma_k$ , which is naturally the case in many practical situations.

Recall that, for the case in which a linear receiver is used to detect the data symbol  $b_k$ , according, i.e., to the decision rule

$$\hat{b}_k = \text{sign} \left[ d_k^T r \right], \quad (5)$$

with  $\hat{b}_k$  the estimate of  $b_k$  and  $d_k$  the  $N$ -dimensional vector representing the receive filter for the user  $k$ , it is easily seen that the SINR  $\gamma_k$  can be written as

$$\gamma_k = \frac{p_k h_k 2 (d_k^T s_k)^2}{\frac{N_0}{2} \|d_k\|^2 + \sum_{i \neq k} p_i h_i 2 (d_k^T s_i)^2} \quad (6)$$

Of related interest is also the mean square error (MSE) for the user  $k$ , which, for a linear receiver, is defined as

$$\text{MSE}_k = E \left\{ (b_k - d_k^T r)^2 \right\} = 1 + d_k^T M d_k - 2 \sqrt{p_k} h_k d_k^T s_k, \quad (7)$$

wherein  $E\{\cdot\}$  denotes statistical expectation and  $M = (S H P H^T S^T + \frac{N_0}{2} I_N)$  is the covariance matrix of the data.

The exact shape of  $P_k(\gamma_k)$  depends on factors such as the modulation and coding type. However, in all cases of relevant interest, it is an increasing function of  $\gamma_k$  with a sigmoidal shape, and converges to unity as  $\gamma_k \rightarrow +\infty$ ; as an example, for binary phase-shift-keying (BPSK) modulation coupled with no channel coding, it is easily shown that

$$P_k(\gamma_k) = \left[ 1 - Q(\sqrt{2\gamma_k}) \right]^M, \quad (8)$$

with  $Q(\cdot)$  the complementary cumulative distribution function of a zero-mean random Gaussian variate with unit variance. A plot of (8) is shown in Fig. 1 for the case  $M = 100$ .

It should be noted that substituting (8) into (4), and, in turn, into (3), leads to a strong incongruence. Indeed, for  $p_k \rightarrow 0$ , we have  $\gamma_k \rightarrow 0$ , but  $P_k$  converges to a small but non-zero value (i.e.  $2^{-M}$ ), thus implying that an unboundedly large utility can be achieved by transmitting with zero power, i.e. not transmitting at all and making blind guesses at the receiver on what data were transmitted. To circumvent this problem, a customary approach [5], [7] is to replace  $P_k$  with

<sup>1</sup>To simplify subsequent notation, we assume that the transmitted power  $p_k$  subsumes also the gain of the transmit and receive antennas.

<sup>2</sup>We assume here, for simplicity, a real channel model; generalization to practical channels, with I and Q components, is straightforward.

<sup>3</sup>Of course there are many other strategies to lower the data error probability, such as for example the use of error correcting codes, diversity exploitation, and implementation of optimal reception techniques at the receiver. Here, however, we are mainly interested to energy efficient data transmission and power usage, so we consider only the effects of varying the transmit power, the receiver and the spreading code on energy efficiency.

an efficiency function, say  $f_k(\gamma_k)$ , whose behavior should approximate as close as possible that of  $P_k$ , except that for  $\gamma_k \rightarrow 0$  it is required that  $f_k(\gamma_k) = o(\gamma_k)$ . The function  $f(\gamma_k) = (1 - e^{-\gamma_k})^M$  is a widely accepted substitute for the true probability of correct packet reception, and in the following we will adopt this model<sup>4</sup>. This efficiency function is increasing and S-shaped, converges to unity as  $\gamma_k$  approaches infinity, and has a continuous first order derivative. Note that we have omitted the subscript “ $k$ ”, i.e. we have used the notation  $f(\gamma_k)$  in place of  $f_k(\gamma_k)$  since we assume that the efficiency function is the same for all the users.

Summing up, substituting (4) into (3) and replacing the probability  $P_k$  with the above defined efficiency function, we obtain the following expression for the  $k$ -th user's utility:

$$u_k = R \frac{L}{M} \frac{f(\gamma_k)}{p_k}, \quad \forall k = 1, \dots, K. \quad (9)$$

Now, based on the utility definition (9), many interesting questions arise concerning how each user may maximize its utility, and how this maximization affects utilities achieved by other users. Likewise, it is natural to question what happens in a non-cooperative setting wherein each user autonomously and selfishly tries to maximize its own utility, with no care for other users utilities. In particular, in this latter situation, is the system able to reach an equilibrium wherein no user is interested in varying its parameters since each action it would take would lead to a decrease in its own utility? Game theory provides means to study these interactions and to provide some useful and insightful answers to these questions.

Initially, game theory was applied in this context mainly as a tool to study non-cooperative scenarios wherein mobile users are allowed to vary their transmit power only (see [4], [5], [6], for example) to maximize utility, and where conventional matched filtering is used at the receiver. Recently, instead, in [7] such an approach has been extended to the cross layer scenario in which each user may vary its power and its uplink linear receiver, i.e. the problem of joint linear multiuser detection optimization and power control for utility maximization has been tackled. In the following, we will go further by considering the case of spreading code choice, power control and linear receiver design for utility maximization. Moreover, the case in which a parametric non-linear decision feedback receiver is used will be considered, and new games wherein optimization of this receiver, spreading code choice and power control is performed jointly in order to maximize utility will be proposed.

### III. NON-COOPERATIVE GAMES WITH LINEAR RECEIVERS

We begin by considering a noncooperative game wherein each user aims to maximizing its own utility by varying its spreading code, its transmit power, and its linear uplink receiver. Formally, the proposed game  $\mathcal{G}$  can be described as the triplet  $\mathcal{G} = [\mathcal{K}, \{S_k\}, \{u_k\}]$ , wherein  $\mathcal{K} = \{1, 2, \dots, K\}$

<sup>4</sup>See Fig. 1 for a comparison between the Probability  $P_k$  and the efficiency function.

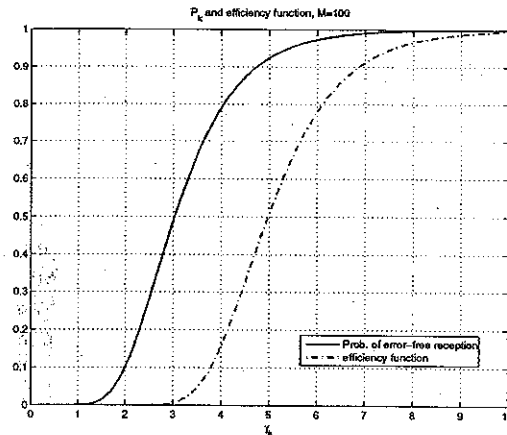


Fig. 1. Comparison of probability of error-free packet reception and efficiency function versus receive SINR and for packet size  $M = 100$ . Note the S-shape of both functions.

is the set of active users participating in the game,  $u_k$  is the  $k$ -th user's utility defined in (9), and

$$S_k = [0, P_{k,\max}] \times \mathcal{R}^N \times \mathcal{R}_1^N, \quad (10)$$

is the set of possible actions (strategies) that user  $k$  can take. It is seen that  $S_k$  is written as the Cartesian product of three different sets, and indeed  $[0, P_{k,\max}]$  is the range of available transmit powers for the  $k$ -th user (note that  $P_{k,\max}$  is the maximum allowed transmit power for user  $k$ ),  $\mathcal{R}^N$ , with  $\mathcal{R}$  the real line, defines the set of all possible linear receive filters, and, finally,

$$\mathcal{R}_1^N = \{d \in \mathcal{R}^N : d^T d = 1\},$$

defines the set of the allowed spreading codes<sup>5</sup> for user  $k$ .

Before proceeding further, it is also convenient to define the concept of *Nash equilibrium*. Let

$$(s_1, s_2, \dots, s_K) \in S_1 \times S_2 \times \dots \times S_K$$

denote a certain strategy  $K$ -tuple for the active users. The point  $(s_1, s_2, \dots, s_K)$  is a Nash equilibrium if for any user  $k$ , we have

$$u_k(s_1, \dots, s_k, \dots, s_K) \geq u_k(s_1, \dots, s_k^*, \dots, s_K),$$

$\forall s_k^* \neq s_k$ . Otherwise stated, at a Nash equilibrium, no user can *unilaterally* improve its own utility by taking a different strategy. A fast reading of this definition might lead to think that at Nash equilibrium users' utilities achieve their maximum values. Actually, this is not the case, since the existence of a Nash equilibrium point does not imply that no other strategy  $K$ -tuple does exist that can lead to an improvement of the utilities of some users while not decreasing the utilities of the remaining ones. These latter strategies are usually

<sup>5</sup>Here we assume that the spreading codes have real entries; the problem of utility maximization with reasonable complexity for the case of discrete-valued entries is a challenging issue that will be considered in the future.

said to be Pareto-optimal [1]. Otherwise stated, at a Nash equilibrium, each user, provided that the other users' strategies do not change, is not interested in changing its own strategy. However, if some sort of cooperation would be available, users might agree to simultaneously switch to a different strategy  $K$ -tuple, so as to improve the utility of some, if not all, active users. In this paper, we will focus on Nash equilibrium points only, since they are the result of non-cooperative games. Moreover, it can be shown, although this is not discussed here due to lack of space, that, for the considered problem, the utilities achieved by Nash-equilibrium points are only slightly smaller than those achieved on the Pareto-optimal frontier of the game.

Summing up, the proposed noncooperative game can be cast as the following maximization problem

$$\max_{S_k} u_k = \max_{p_k, d_k, s_k} u_k(p_k, d_k, s_k), \quad \forall k = 1, \dots, K. \quad (11)$$

Given (9), the above maximization can be also written as

$$\max_{p_k, d_k, s_k} \frac{f(\gamma_k(p_k, d_k, s_k))}{p_k}, \quad \forall k = 1, \dots, K. \quad (12)$$

Moreover, since the efficiency function is monotone and non-decreasing, we also have

$$\max_{p_k, d_k, s_k} \frac{f(\gamma_k(p_k, d_k, s_k))}{p_k} = \max_{p_k} \frac{f\left(\max_{d_k, s_k} \gamma_k(p_k, d_k, s_k)\right)}{p_k}, \quad (13)$$

i.e. we can first take care of SINR maximization with respect to spreading codes and linear receivers, and then focus on maximization of the resulting utility with respect to transmit power.

With regard to this latter point, recall that, if a linear Minimum MSE (MMSE) receiver is used, the following relation can be shown to hold [9]

$$\text{TMSE} = \sum_{i=1}^K \text{MSE}_i = \sum_{i=1}^K \frac{1}{1 + \gamma_i}, \quad (14)$$

wherein TMSE is the total MSE. Otherwise stated, among linear receivers, the MMSE receiver is the one that maximizes the SINR vector  $(\gamma_1, \dots, \gamma_K)$ . Now assume that we wish to minimize the MSE for each user by varying not only the receiver, but also the spreading code. This problem has been considered in [8], [10], [11]; in particular, if we let  $D = [d_1, \dots, d_K]$  and denote by  $(\cdot)^+$  Moore-Penrose pseudoinversion, it has been therein shown that the sum of the MSE's of all the users admits a unique global optimum, and that the iterations

$$\begin{aligned} d_i &= \sqrt{p_i} h_i \left( S H P H^T S^T + \frac{N_0}{2} I_N \right)^{-1} s_i \quad \forall i = 1, \dots, K \\ s_i &= \sqrt{p_i} h_i \left( p_i h_i 2 D D^T + \mu_i I_N \right)^+ d_i \quad \forall i = 1, \dots, K \end{aligned} \quad (15)$$

admit a unique stable fixed point that is the global minimizer of the total MSE. In the above relations,  $\mu_i$  should be set so

that  $\|s_i\| = 1$ . No details are given in [8] on how this could be done in an efficient way, so in the Appendix we outline a procedure for efficiently finding the value of  $\mu_i$  ensuring the constraint  $\|s_i\| = 1$ .

Now, it is natural to ask if minimization of the total MSE with respect to both linear receivers and spreading codes still maximizes the user's SINR's. We can thus state the following result.

**Lemma 1:** Let  $\bar{S}$  and  $\bar{D}$  be the spreading code matrix and the linear receiver matrix that jointly achieve the global minimum of the total MSE. Then, no strategy of spreading codes and decoder can be found to increase the SINR of one or more users without decreasing the SINR of at least one other user.

**Proof:** If  $\bar{S}$  and  $\bar{D}$  are the global minimizers of the MSE, then  $\bar{D}$  contains the MMSE receivers resulting from the spreading codes of  $\bar{S}$ . Denote by  $\{\gamma_i(\bar{S}, \bar{D})\}_{i=1}^K$  the SINR values achieved by the matrices  $\bar{S}$  and  $\bar{D}$ . Assume now that there exists a spreading code matrix  $S^* \neq \bar{S}$  such that  $\gamma_i(S^*, \bar{D}) > \gamma_i(\bar{S}, \bar{D})$ , for at least one  $i \in \{1, \dots, K\}$  and  $\gamma_j(S^*, \bar{D}) \geq \gamma_j(\bar{S}, \bar{D})$  for  $j \neq i$ . If this is the case, we can make an MMSE update and obtain the matrix  $D^*$  of the MMSE receivers corresponding to the codes in  $S^*$ . For a given set of spreading codes, using the MMSE receiver always yields a maximization of the SINR and a minimization of the MSE. We thus have  $\gamma_i(S^*, D^*) > \gamma_i(\bar{S}, \bar{D})$ , and  $\gamma_j(S^*, D^*) \geq \gamma_j(\bar{S}, \bar{D})$ ,  $\forall j \neq i$ . Consequently, given relation (14), we have

$$\text{TMSE}(S^*, D^*) < \text{TMSE}(\bar{S}, \bar{D}),$$

which contradicts the starting assumptions that  $\bar{S}$  and  $\bar{D}$  are the global minimizers of the MSE. ■

We are now ready to express our result on the non-cooperative game for spreading code optimization, linear receiver design and power control.

**Proposition 1:** The non-cooperative game defined in (11) admits a unique Nash equilibrium point  $(p_k^*, d_k^*, s_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $s_k^*$  and  $d_k^*$  are the unique  $k$ -th user spreading code and receive filter<sup>6</sup> resulting from iterations (15). Denote by  $\gamma_k^*$  the corresponding SINR.
- $p_k^* = \min\{\bar{p}_k, P_{k, \max}\}$ , with  $\bar{p}_k$  the  $k$ -th user transmit power such that the  $k$ -th user maximum SINR  $\gamma_k^*$  equals  $\bar{\gamma}$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  the derivative of  $f(\gamma)$ .

**Proof:** The proof is a generalization of the one provided in [7] and so is only briefly sketched here. Since  $\partial \gamma_k / \partial p_k = \gamma_k / p_k$ , it is easily seen that each user's utility is maximized if each user is able to achieve the SINR  $\bar{\gamma}$ , that is the unique<sup>7</sup> solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ . By Lemma 1, running iterations (15) until convergence is reached provides the set of spreading codes and MMSE receivers that maximize the SINRs for all the users. As a consequence, the utility of

<sup>6</sup>Actually the linear receive filter is unique up to a positive scaling factor.

<sup>7</sup>Uniqueness of  $\bar{\gamma}$  is ensured by the fact that the efficiency function is S-shaped [12].

each user is maximized by adjusting transmit powers so that the optimized (with respect to spreading codes and linear receivers) SINRs equal  $\bar{\gamma}$ .

So far, we have shown how to set the transmit power, spreading code and receiver design to maximize utility at the Nash equilibrium. It remains to be shown that a Nash equilibrium exists. Luckily, we can use the same arguments of [5] and state that a unique Nash equilibrium point exists since each user's utility function is quasi-concave<sup>8</sup> in the transmit power  $p_k$  and since the efficiency function is S-shaped. ■

#### IV. NON-COOPERATIVE GAMES WITH NONLINEAR DECISION-FEEDBACK RECEIVERS

Consider now the case in which a non-linear decision feedback receiver is used at the receiver. We assume that the users are indexed according to a non-increasing sorting of their channel gains, i.e. we assume that  $h_1 > h_2 > \dots, h_K$ . We consider a serial interference cancellation (SIC) receiver wherein detection of the symbol from the  $k$ -th user is made according to the following rule

$$\hat{b}_k = \text{sign} \left[ d_k^T \left( r - \sum_{j < k} \sqrt{p_j} h_j \hat{b}_j s_j \right) \right] \quad (16)$$

Otherwise stated, when detecting a certain symbol, the contribution from the data symbols that have been already detected is subtracted from the received data. If past decisions are correct, users that are detected later enjoy a considerable reduction of multiple access interference, and indeed the SINR for user  $k$ , under the assumption of correctness of past decisions, is written as

$$\gamma_k = \frac{p_k h_k 2(d_k^T s_k) 2}{\frac{N_0}{2} \|d_k\|^2 + \sum_{j > k} p_j h_j 2(d_k^T s_j) 2} \quad (17)$$

A considerable amount of literature exists on decision feedback receivers, and many detectors of this kind have been proposed and analyzed. Here, our goal is just to show that non-linear receivers coupled with spreading code optimization and power control can bring remarkable performance advantages with respect to linear receivers. As a consequence, we consider only the decision rule (16) and introduce noncooperative games built on that, with no further optimization. As an example, receiver (16) might be optimized with respect to the users' detection order, or by using properly distorted versions of the signal to be subtracted; these issues will not be considered here due to lack of space. ■

Now, given receiver (16) and the SINR expression (17), we consider the problems of utility maximization with respect to the transmit power, spreading code choice, and receivers  $d_1, \dots, d_K$ . To begin with, let us neglect spreading code optimization and consider the problem

$$\max_{p_k, d_k} \frac{f(\gamma_k(p_k, d_k))}{p_k}, \quad \forall k = 1, \dots, K. \quad (18)$$

<sup>8</sup>A function is quasi-concave if there exists a point below which the function is nondecreasing, and above which the function is nonincreasing.

The following result can be shown to hold.

**Proposition 2:** Define  $S_k = [s_k, \dots, s_K]$ ,  $P_k = \text{diag}(p_k, \dots, p_K)$  and  $H_k = \text{diag}(h_k, \dots, h_K)$ . The non-cooperative game defined in (18) admits a unique Nash equilibrium point  $(p_k^*, d_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $d_k^* = \sqrt{p_k} h_k (S_k H_k P_k H_k^T S_k^T + \frac{N_0}{2} I_N)^{-1} s_k$  is the unique  $k$ -th user receive filter<sup>9</sup> that maximize the user  $k$  SINR  $\gamma_k$  given in (17). Denote  $\gamma_k^* = \max_{d_k} \gamma_k$ .
- $p_k^* = \min\{\bar{p}_k, P_{k, \max}\}$ , with  $\bar{p}_k$  the  $k$ -th user transmit power such that the  $k$ -th user maximum SINR  $\gamma_k^*$  equals  $\bar{\gamma}$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  the derivative of  $f(\gamma)$ .

**Proof:** The proof is omitted here due to lack of space. It can be constructed along the same lines as that of Proposition 1. ■

Consider, finally, the maximization

$$\max_{p_k, d_k, s_k} \frac{f(\gamma_k(p_k, d_k, s_k))}{p_k}, \quad \forall k = 1, \dots, K. \quad (19)$$

The existence and uniqueness of a Nash equilibrium for this game is guaranteed by the following result.

**Proposition 3:** Define  $D_k = [d_1, \dots, d_k]$ . The non-cooperative game defined in (19) admits a unique Nash equilibrium point  $(p_k^*, s_k^*, d_k^*)$ , for  $k = 1, \dots, K$ , wherein

- $d_k^*$  and  $s_k^*$  are the unique stable fixed points of the iterations  $d_k = \sqrt{p_k} h_k (S_k H_k P_k H_k^T S_k^T + \frac{N_0}{2} I_N)^{-1} s_k$  and  $s_k = \sqrt{p_k} h_k (p_k h_k 2 D_k D_k^T + \mu_k I_N)^{-1} d_k$ ,  $\forall k = 1, \dots, K$  and with  $\mu_k$  such that  $\|s_k\| = 1$ . Denote by  $\gamma_k^*$  the  $k$ -th user's SINR resulting from the choices  $s_k = s_k^*$  and  $d_k = d_k^*$ .
- $p_k^* = \min\{\bar{p}_k, P_{k, \max}\}$ , with  $\bar{p}_k$  the  $k$ -th user transmit power such that the  $k$ -th user maximum SINR  $\gamma_k^*$  equals  $\bar{\gamma}$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  the derivative of  $f(\gamma)$ .

**Proof:** The proof is omitted here due to lack of space. ■

#### V. NUMERICAL RESULTS

In this section we illustrate some simulation results that give insight into the performance of the proposed non-cooperative games. We contrast here the performance of the noncooperative game discussed in [7] with that of the games proposed here. We consider an uplink DS/CDMA system with processing gain  $N = 7$ , and assume that the packet length is  $M = 120$ . For this value of  $M$  the equation  $f(\gamma) = \gamma f'(\gamma)$  can be shown to admit the solution  $\bar{\gamma} = 6.689 = 8.25\text{dB}$ . A single-cell system is considered, wherein users may have random positions with a distance from the AP ranging from 10m to 500m. The channel coefficient  $h_k$  for the generic  $k$ -th user is assumed to be Rayleigh distributed with mean equal to  $d_k^{-2}$ , with  $d_k$  being the distance of user  $k$  from the AP<sup>10</sup>. We

<sup>9</sup>Uniqueness here means up to a positive scaling factor.

<sup>10</sup>Note that we are here assuming that the power path losses are proportional to the fourth power of the path length, which is reasonable in urban cellular environments.

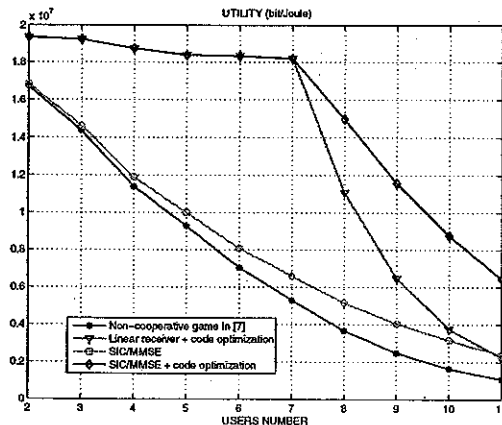


Fig. 2. Achieved average utility versus number of active users for the proposed noncooperative games and for the game in reference [7]. The system processing gain is  $N = 7$ .

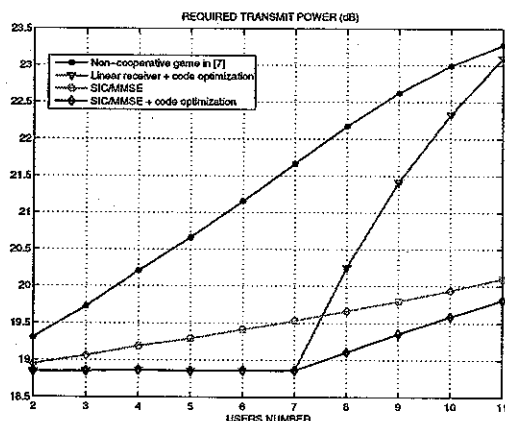


Fig. 3. Average transmit power versus number of active users for the proposed noncooperative games and for the game in reference [7]. The system processing gain is  $N = 7$ .

take the ambient noise level to be  $N_0 = 10^{-9}$  W/Hz, while the maximum allowed power  $P_{k,\max}$  is 25dB. We present the results of averaging over 104 independent realizations for the users locations, fading channel coefficients and starting set of spreading codes. More precisely, for each iteration we randomly generate an  $N \times K$ -dimensional spreading code matrix with entries in the set  $\{-1/\sqrt{N}, 1/\sqrt{N}\}$ ; this matrix is then used as the starting point for the games that include spreading code optimization, and as the spreading code matrix for the games that do not perform spreading code optimization.

Figures 2 - 4 report the achieved average utility (measured in bits/Joule), the average user transmit power and the average achieved SINR at the receiver output for the game in [7] and for the three non-cooperative games considered in this paper. Inspecting the curves, the following conclusions can be drawn. First of all, it is seen that all of the proposed games outperform the one proposed in [7]: of course this result can be easily justified by noting that the proposed games can take advantage

of the optimization of the spreading codes and of the superior performance that non-linear receivers provide over linear ones. As an example, it is seen that for a system with  $K = N = 7$  the game with SIC/MMSE plus spreading code optimization achieves a utility that is more than 3 times larger than that achieved by the game in [7] and with a simultaneous average transmit power saving of almost 3dB.

For  $K \leq N$ , a very substantial performance gain can be obtained by resorting to spreading code optimization; indeed, when  $K \leq N$ , users can be given orthogonal spreading codes, so that the multiaccess channel reduces to a superposition of  $K$  separate single-user AWGN channels. Obviously, in this situation the spreading code optimization algorithms converge to a set of orthogonal codes, and this explains the performance gains reported in the figures. Interestingly, for  $K \leq N$  the performance of the linear MMSE and of the SIC/MMSE receivers with spreading code optimization coincide: this is an indirect confirmation that in this case the steady-state spreading codes are orthogonal, since in this case no distinction occurs between the SIC/MMSE receiver and the linear one. In the oversaturated region (i.e. for  $K > N$ ), instead, the merits of the non-linear SIC/MMSE can be clearly seen. Indeed, in this situation the spreading codes, whether optimized or not, are linearly dependent, and this leads to a severe performance degradation for any linear processing. In this region SIC/MMSE plus spreading code optimization is thus the best option, followed by SIC/MMSE with no spreading code optimization. Note that there is a crossing around  $K = 11$  between the performance of the MMSE receiver with spreading code optimization and that of the SIC/MMSE with no spreading code optimization, revealing that for lightly loaded systems much can be gained through spreading code optimization, while for heavily loaded systems the most significant gains come from the use of non-linear processing. It is also seen from Fig. 4 that receivers achieve on the average an output SINR that is smaller than the target SINR  $\bar{\gamma}$ : indeed, due to fading and distance path losses, achieving the target SINR would require for some users a transmit power larger than the maximum allowed power  $P_{k,\max}$ , and so these users are not able to achieve the optimal target SINR. As a confirmation of this, in Fig. 5 we report the fraction of users transmitting at the maximum power: it is seen here that even for the SIC/MMSE receiver with spreading code optimization this fraction is larger than 0.1.

## VI. CONCLUSION

In this paper the cross-layer issue of joint power control, spreading code optimization and receiver design for wireless data networks has been addressed using a game-theoretic framework. Building on [7], we have proposed a more general framework wherein also spreading code optimization and non-linear decision feedback multiuser receivers can be used to further increase the energy efficiency of CDMA-based wireless networks. It has been shown that spreading code optimization in non-overloaded system, and non-linear reception techniques in overloaded systems, bring remarkable performance gains.

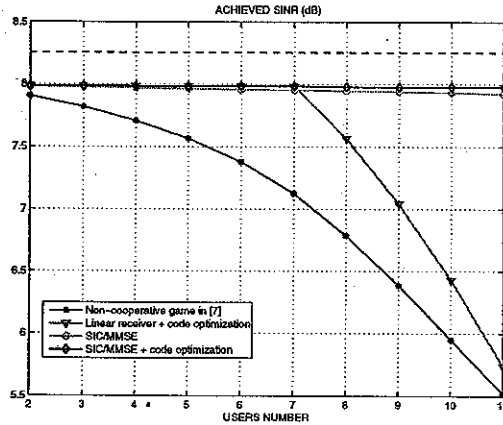


Fig. 4. Achieved average output SINR versus number of active users for the proposed noncooperative games and for the game in reference [7]. The system processing gain is  $N = 7$ .

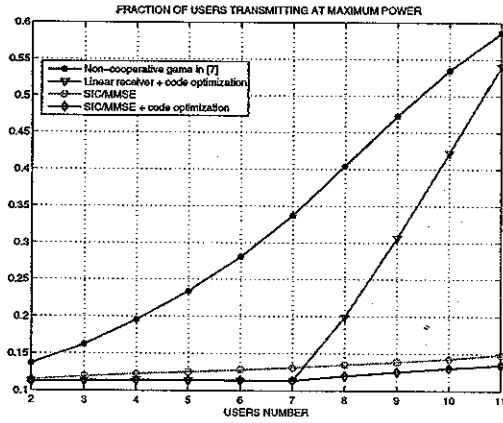


Fig. 5. Average fraction of users transmitting at their maximum allowed power versus number of active users for the proposed noncooperative games and for the game in reference [7]. The system processing gain is  $N = 7$ .

The authors' current research is focused on the development and the analysis of adaptive algorithms able to implement the said games without prior knowledge of the fading channel coefficients.

#### ACKNOWLEDGMENT

This research in this paper was supported by the Italian National Research Council (CNR), by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252, and by the U. S. Defense Advanced Research Projects Agency under Grant No. HR0011-06-1-0052. The authors wish to thank Dr. Husheng Li for insightful comments on a preliminary version of this paper.

#### APPENDIX

Given the relation

$$s_k = \sqrt{p_k} h_k (p_k h_k 2DD^T + \mu_k I_N)^+ d_k \quad (20)$$

we show here how to choose the constant  $\mu_k$  so that  $\|s_k\| = 1$ . Let  $UAU^T$  be the eigendecomposition of the matrix  $p_k h_k 2DD^T$ . Obviously,  $U$  is an orthonormal matrix whose columns are the eigenvectors of  $p_k h_k 2DD^T$ , and  $\Lambda$  is the corresponding diagonal eigenvalue matrix. Note that some of these eigenvalue will be zero for  $K < N$ . Now, letting  $u_i$  and  $\lambda_i$  denote the  $i$ -th column of  $U$  and the  $i$ -th diagonal element of  $\Lambda$ , respectively, and

$$z(\lambda_i, \mu_k) = \begin{cases} \frac{1}{\lambda_i + \mu_k} & \text{if } \lambda_i + \mu_k \neq 0 \\ 0 & \text{if } \lambda_i + \mu_k = 0, \end{cases} \quad (21)$$

it is easy to show that (20) can be rewritten as

$$s_k = \sqrt{p_k} \sum_{i=1}^N z(\lambda_i, \mu_k) u_i u_i^T d_k. \quad (22)$$

From (22) it is seen that, as  $\mu_k \rightarrow +\infty$ ,  $\|s_k\| \rightarrow 0$ , thus implying that there exists a finite constant  $Q_u$  such that  $\|s_k\| < 1$  for any  $\mu_k \geq Q_u$ . Now, let  $\lambda_m = \min_i \lambda_i$  (note that  $\lambda_m$  may be 0 if  $K < N$  or in general if  $DD^T$  is not of full rank). It is easy to show that, as  $\mu_k \rightarrow \lambda_m^+$ ,  $\|s_k\| \rightarrow +\infty$ . Accordingly, there exists a finite constant  $Q_l > \lambda_m$  such that  $\|s_k\| > 1$  for  $\mu_k \in [\lambda_m, Q_l]$ . Since  $\|s_k\|$  is monotonically decreasing for  $\mu_k \in [Q_l, Q_u]$  and since  $\|s_k\| > 1$  for  $\mu_k = Q_l$  and  $\|s_k\| < 1$  for  $\mu_k = Q_u$ , there exists just one value of  $\mu_k$ , say  $\mu_k^*$ , such that  $\|s_k\| = 1$  for  $\mu_k = \mu_k^*$ . The value of  $\mu_k^*$  can be found using standard methods.

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# Power Control Algorithms for CDMA Networks Based on Large System Analysis

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**Abstract**—Power control is a fundamental task accomplished in any wireless cellular network; its aim is to set the transmit power of any mobile terminal, so that each user is able to achieve its own target SINR. While conventional power control algorithms require knowledge of a number of parameters of the signal of interest and of the multiaccess interference, in this paper it is shown that in a large CDMA system much of this information can be dispensed with, and effective distributed power control algorithms may be implemented with very little information on the user of interest. An uplink CDMA system subject to flat fading is considered with a focus on the cases in which a linear MMSE receiver and a non-linear MMSE serial interference cancellation receiver are adopted; for the latter case new formulas are also given for the system SINR in the large system asymptote. Experimental results show an excellent agreement between the performance and the power profile of the proposed distributed algorithms and that of conventional ones that require much greater prior knowledge.

## I. INTRODUCTION

In multiuser wireless communication systems, mobile users vary their transmit powers so as to counteract channel gain variations and achieve their target Signal-to-Interference plus Noise Ratios (SINRs); this task is usually referred to as power control [1]. In code division multiple access (CDMA) wireless systems, power control, possibly coupled with the use of multiuser detectors, is thus used in the uplink to combat the near-far effect, to manage interference levels, and to minimize the overall power radiated by the system. A considerable amount of work has been done on power control algorithms for cellular networks, see, e.g. [1]–[3], to cite a few; these algorithms, based on measurements taken at both the mobile station (MS) and at the base station (BS), give as output the transmit power for each terminal. In power control procedures, estimates of several parameters such as the channel gains and the SINR for each user are usually needed, or, alternatively, recursive algorithms are adopted (see, e.g. [3]), which however are affected by slow convergence speed and excess steady-state error.

In recent years a new mathematical tool has emerged in the analysis of CDMA systems, i.e. the so-called large system analysis, first introduced in [4]. In short, [4] has revealed that, in a CDMA system whose processing gain and number of users both increase without bound with their ratio fixed, and with randomly chosen unit-norm spreading codes, the SINR at the output of a linear minimum mean square error (MMSE)

receiver converges in probability to a non-random constant. In particular, denoting by  $K$  the number of active users, by  $N$  the system processing gain, by  $\sigma^2$  the additive thermal noise power spectral density (PSD) level, and by  $E_P[\cdot]$  the expectation with respect to the limiting empirical distribution  $F$  of the received powers of the interferers, the SINR of the MMSE receiver for the  $k$ -th user, say  $\gamma_k$ , converges in probability as  $K, N \rightarrow \infty$ ,  $K/N = \alpha = \text{constant}$ , to  $\gamma_k^*$  the unique solution of the equation

$$\gamma_k^* = \frac{P_k}{\sigma^2 + \alpha E_P \left[ \frac{P P_k}{P_k + P_k^*} \right]}, \quad (1)$$

with  $P_k$  the received power for the  $k$ -th user. Interestingly, the limiting SINR depends only on the limiting empirical distribution of the received powers of the interferers, the load  $\alpha$ , the thermal noise level and the transmit power of the user of interest, while being independent of the actual realization of the received powers of the interferers and of the spreading codes of the active users. Large system analysis is now a well-established mathematical tool for the design and analysis of communication systems (see, e.g., [5], [6], to cite a few).

In this paper, we show how large system analysis can be used to design distributed power control algorithms that need very little prior information (i.e. the channel gain for the user of interest) to be implemented, for both the cases in which linear MMSE detection and non-linear interference cancellation MMSE detection are used at the receiver. The contributions of this paper can thus be summarized as follows.

- Assuming that linear MMSE detection is used at the receiver, we propose a new distributed power control algorithm requiring little prior information and based on large system analysis.
- Extending the approach in [4], we give an expression for the limiting SINR in the case in which non-linear serial interference cancellation MMSE (SIC/MMSE) detection is adopted at the receiver.
- Assuming that SIC/MMSE detection is used at the receiver, we propose a new distributed power control algorithm based on large system approximations.
- We show how the proposed algorithms can be included in utility-maximizing non-cooperative games in order to achieve energy-efficiency in wireless data networks [7].

Our numerical results will show that the proposed algorithms are effective and are able to approach very closely the power profile predicted by much more complex and information-demanding power control algorithms.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

Consider the uplink of a  $K$ -user synchronous, single-cell, direct-sequence code division multiple access (DS/CDMA) system with processing gain  $N$  and subject to flat fading. After chip-matched filtering and sampling at the chip-rate, the  $N$ -dimensional received data vector, say  $\mathbf{r}$ , corresponding to one symbol interval, can be written as

$$\mathbf{r} = \sum_{k=1}^K \sqrt{p_k} h_k b_k \mathbf{s}_k + \mathbf{n}, \quad (2)$$

wherein  $p_k$  is the transmit power of the  $k$ -th user<sup>1</sup>,  $b_k \in \{-1, 1\}$  is the information symbol of the  $k$ -th user, and  $h_k$  is the real<sup>2</sup> channel gain between the  $k$ -th user transmitter and the base station; the actual value of  $h_k$  depends on both the distance of the  $k$ -th user's mobile from the base station and the channel fading fluctuations. The  $N$ -dimensional vector  $\mathbf{s}_k$  is the spreading code of the  $k$ -th user; we assume that the entries of  $\mathbf{s}_k$  are binary-valued and that  $\mathbf{s}_k^T \mathbf{s}_k = \|\mathbf{s}_k\|^2 = 1$ , with  $(\cdot)^T$  denoting transpose. Finally,  $\mathbf{n}$  is the thermal noise vector, which we assume to be a zero-mean white Gaussian random process with covariance matrix  $\sigma^2 \mathbf{I}_N$ , with  $\mathbf{I}_N$  the identity matrix of order  $N$ .

Given the system model (2), a number of strategies are available to detect the data symbols  $b_1, \dots, b_K$ ; in the following we briefly review the linear MMSE receiver and the non-linear SIC/MMSE receiver.

### A. Linear MMSE detection

Consider the case in which a linear receiver is used to detect the data symbol  $b_k$ , according, i.e., to the decision rule  $\hat{b}_k = \text{sign}[\mathbf{d}_k^T \mathbf{r}]$ , with  $\hat{b}_k$  the estimate of  $b_k$  and  $\mathbf{d}_k$  the  $N$ -dimensional vector representing the receive filter for the user  $k$ . Then, it is easily seen that the linear MMSE receiver is the one corresponding to the choice  $\mathbf{d}_k = \sqrt{p_k} h_k (\mathbf{S} \mathbf{H} \mathbf{P} \mathbf{H}^T \mathbf{S}^T + \sigma^2 \mathbf{I}_N)^{-1} \mathbf{s}_k$ , wherein  $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_K]$  is the  $N \times K$ -dimensional spreading-code matrix, and  $\mathbf{P}$  and  $\mathbf{H}$  are  $K \times K$ -dimensional diagonal matrices, whose diagonals are  $[p_1, \dots, p_K]$  and  $[h_1, \dots, h_K]$ , respectively. For linear detectors it is also meaningful to define the output SINR, which, for the  $k$ -th user is written as

$$\gamma_k = \frac{p_k h_k^2 (\mathbf{d}_k^T \mathbf{s}_k)^2}{\sigma^2 \|\mathbf{d}_k\|^2 + \sum_{i \neq k} p_i h_i^2 (\mathbf{d}_k^T \mathbf{s}_i)^2} \quad (3)$$

<sup>1</sup>To simplify subsequent notation, we assume that the transmitted power  $p_k$  subsumes also the gain of the transmit and receive antennas.

<sup>2</sup>For simplicity, we assume a real channel model; however, generalization to practical channels with I and Q components is straightforward.

### B. Non-linear SIC/MMSE detection

Consider now the case in which non-linear decision feedback detection is used at the receiver. We assume that the users are indexed according to a non-increasing sorting of their channel gains, i.e. we assume that  $h_1 > h_2 > \dots, h_K$ . We consider a serial interference cancellation (SIC) receiver wherein detection of the symbol from the  $k$ -th user is made according to the rule  $\hat{b}_k = \text{sign}[\mathbf{d}_k^T \mathbf{r}_k]$ , wherein  $\mathbf{r}_k = \mathbf{r} - \sum_{j < k} \sqrt{p_j} h_j \hat{b}_j \mathbf{s}_j$ .

Otherwise stated, when detecting a certain symbol, the contributions from the data symbols that have already been detected are subtracted from the received data. The output SINR for user  $k$ , under the assumption of correctness of past decisions, is now written as

$$\gamma_k = \frac{p_k h_k^2 (\mathbf{d}_k^T \mathbf{s}_k)^2}{\sigma^2 \|\mathbf{d}_k\|^2 + \sum_{j > k} p_j h_j^2 (\mathbf{d}_k^T \mathbf{s}_j)^2} \quad (4)$$

Upon defining  $\mathbf{S}_k = [\mathbf{s}_k, \dots, \mathbf{s}_K]$ ,  $\mathbf{P}_k = \text{diag}(p_k, \dots, p_K)$  and  $\mathbf{H}_k = \text{diag}(h_k, \dots, h_K)$ , it is easy to show that SIC/MMSE detection corresponds to the choice  $\mathbf{d}_k = \sqrt{p_k} h_k (\mathbf{S}_k \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{S}_k^T + \sigma^2 \mathbf{I}_N)^{-1} \mathbf{s}_k$ .

### C. Problem statement

Given the data model (2), we are interested in the following problem: find the transmit power  $p_k \in [0, P_{\max}]$  for each user  $k$ , so that the SINR  $\gamma_k$  equals a given target value  $\bar{\gamma}$ , with  $P_{\max}$  the maximum power that each user in the system is allowed to transmit. Note that this problem finds numerous applications. As an example, in circuit-switched wireless cellular voice communications, wherein the primary goal is signal intelligibility, the SINR is required to be always above a given intelligibility threshold [1]. In wireless packet-switched networks, instead, the primary goal may be to maximize the system throughput, or, if battery-life of the mobile terminals is a dominant issue, the system throughput for each unit or energy drained from the battery [2], [7]. In all cases, however, it can be shown that this goal translates into the requirement that each user's SINR equals at least a certain value.

In the sequel, we show how large system analysis leads to power control algorithms that may be implemented in a distributed fashion and that require knowledge of the channel for the user of interest only.

## III. POWER CONTROL FOR LINEAR MMSE DETECTION

As an introductory step in our algorithm, we begin by illustrating a simple power control algorithm derived from [4]. We have seen that in a large CDMA system the  $k$ -th user's SINR converges in probability to the solution to Eq. (1). Heuristically, this means that in a large system, and embracing the notation of the previous section, the SINR  $\gamma_k$  is deterministic and approximately satisfies

$$\gamma_k \approx \frac{h_k^2 p_k}{\sigma^2 + \frac{1}{N} \sum_{j \neq k} \frac{h_k^2 h_j^2 p_k p_j}{h_k^2 p_k + h_j^2 p_j \gamma_k}} \quad (5)$$



Now, as noted in [4], if all the users must achieve the same common target SINR  $\bar{\gamma}$ , it is reasonable to assume that they are to be received with the same power, i.e. the condition  $h_1^2 p_1 = h_2^2 p_2 = \dots h_K^2 p_K = P_R$ , is to be fulfilled. Substituting the above constraint in (5) and equating (5) to  $\bar{\gamma}$  it is straightforward to come up with the following relation

$$P_R = \frac{\bar{\gamma} \sigma^2}{1 - \frac{\bar{\gamma}}{1+\bar{\gamma}} \alpha} \Rightarrow p_k = \frac{1}{h_k^2} \frac{\bar{\gamma} \sigma^2}{1 - \frac{\bar{\gamma}}{1+\bar{\gamma}} \alpha}, \quad (6)$$

wherein, we recall,  $\alpha = K/N$ , and the relation  $\alpha < 1 + 1/\bar{\gamma}$  must hold. Eq. (6), which descends from eq. (16) in [4], gives a simple power control algorithm that permits setting the transmitted power for each user based on the knowledge of the channel gain for the user of interest only. The above algorithm, however, does not take into account the situation in which, due to fading and path losses, some users end up transmitting at their maximum power without achieving the target SINR, and indeed our numerical results to be shown in the sequel will prove the inability of (6) to predict with good accuracy the actual power profile for the active users.

In order to circumvent this drawback, we first recall that in [8] (see also [9]) the following result has been shown:

**Lemma:** Denote by  $F(\cdot)$  the cumulative distribution function (CDF) of the squared fading coefficients  $h_i^2$ , and by  $[h_{[1]}^2, h_{[2]}^2, \dots, h_{[K]}^2]$  the vector of the users' squared fading coefficients sorted in non-increasing order. Then we have that  $h_{[\ell]}^2$  converges in probability (as  $K \rightarrow \infty$ ) to  $F^{-1}(\frac{K-\ell}{K})$ ,  $\forall \ell = 1, \dots, K$ .

The above lemma states that if we sort a large number of identically distributed random variates, we obtain a vector that is approximately equal to the uniformly sampled version of the inverse of the common CDF of the random variates. Accordingly, in a large CDMA system each user may individually build a rough estimate of the fading coefficients in the network and thus will be able to predict the number of users, say  $u_2$ , that possibly will wind up transmitting at the maximum power. Indeed, since, according to (6) each user is to be received with a power  $P_R$ , the estimate  $u_2$  of the number of users transmitting at the maximum power is given by

$$u_2 = \sum_{i=1}^K u \left( \frac{\bar{\gamma} \sigma^2}{F^{-1}(\frac{K-i}{K}) (1 - \frac{\bar{\gamma}}{1+\bar{\gamma}} \alpha)} - P_{\max} \right), \quad (7)$$

with  $u(\cdot)$  the step-function. It is also natural to assume that the users transmitting at  $P_{\max}$  will be the ones with the smallest channel coefficients, i.e. the squared channel gains of the users transmitting at the maximum power are well approximated by the samples  $F^{-1}(\frac{K-\ell}{K})$ , with  $\ell = K - u_2 + 1, \dots, K$ . As a consequence, the generic  $k$ -th user will be affected by  $u_1 = K - u_2$  users that are received with power  $P_R$  (these are the  $u_1$  users with the strongest channel gains and that are able to achieve the target SINR  $\bar{\gamma}$ ), and by  $u_2$  users that are received with power  $P_{\max} F^{-1}(\frac{K-\ell}{K})$ , with  $\ell = K - u_2 + 1, \dots, K$ . Denoting by  $P_k$  the received power for the  $k$ -th user, Eq. (5)

can be now written as

$$\gamma_k = \frac{P_k}{\sigma^2 + \frac{u_1}{N} \frac{P_k P_R}{P_k + P_R \gamma_k} + \frac{1}{N} \sum_{i=K-u_2+1}^K \frac{P_k P_{\max} F^{-1}(\frac{K-i}{K})}{P_k + P_{\max} F^{-1}(\frac{K-i}{K}) \gamma_k}} \quad (8)$$

Now, assuming for the moment that user  $k$  is able to achieve its target SINR, i.e. that  $P_k = P_R$ , we can make the approximation  $\frac{P_k P_R}{P_k + P_R \gamma_k} \approx \frac{P_k}{1 + \gamma_k}$ , whereby equating (8) to the target SINR  $\bar{\gamma}$  we have

$$\frac{P_k}{\sigma^2 + \frac{u_1}{N} \frac{P_k}{1 + \bar{\gamma}} + \frac{1}{N} \sum_{i=K-u_2+1}^K \frac{P_k P_{\max} F^{-1}(\frac{K-i}{K})}{P_k + P_{\max} F^{-1}(\frac{K-i}{K}) \bar{\gamma}}} = \bar{\gamma}. \quad (9)$$

The above relation can now be solved numerically in order to determine the receive power  $P_k$  for the  $k$ -th user<sup>3</sup>. Finally, the actual transmit power for the  $k$ -th user is set according to the rule

$$p_k = \min \{ P_k / h_k^2, P_{\max} \}. \quad (10)$$

The proposed algorithm may be summarized as follows. First, the number of users transmitting at the maximum power is estimated according to (7). Then, the desired receive power for each user is computed by solving (9). Finally, the transmit power for the  $k$ -th user is determined according to relation (10). Note that this algorithm requires knowledge only of the channel gain for the user of interest.

#### IV. POWER CONTROL FOR SIC/MMSE DETECTION

Let us now consider the case in which non-linear SIC/MMSE detection is used at the receiver, and let us thus assume that users are ordered according to a non-increasing power profile. In this case the following theorem can be proved.

**Theorem:** Let  $\gamma_k$  be the (random) SINR of the SIC/MMSE receiver for the  $k$ -th user; let  $P_k$  be the received power for the  $k$ -th user, and assume that previously detected symbols have been perfectly cancelled. As  $K, N \rightarrow +\infty$ , with  $K/N = \alpha$ ,  $\gamma_k$  converges in probability to  $\gamma_k^*$ , the unique solution of the equation

$$\gamma_k^* = \frac{P_k}{\sigma^2 + \alpha_k E_{P|P < P_k} \left[ \frac{P P_k}{P_k + P \gamma_k^*} \right]}, \quad (11)$$

with  $\alpha_k = (K - k + 1)/N$  and  $E_{P|P < P_k}[\cdot]$  denoting expectation with respect to the empirical distribution of the received powers not larger than  $P_k$ .

**Proof:** The proof is omitted for the sake of brevity.

To corroborate the statement of the above theorem, in Fig. 1 we report the asymptotic SINR and 100 actual realizations of the SINR corresponding to random realizations of the spreading codes and of the received powers, assumed to follow a Rayleigh distribution. A system with processing gain  $N = 256$  has been considered here, and it is seen that the random SINR realizations are spread around their asymptotic value.

<sup>3</sup> Actually this equation gives the desired receive power for each user, and thus in a centralized power control algorithm needs to be solved only once.

Using the notation of Section II, a heuristic reformulation of the above theorem states that in a large system the  $k$ -th user's SINR  $\gamma_k$  is deterministic and approximately satisfies the equation

$$\gamma_k \approx \frac{h_k^2 p_k}{\sigma^2 + \frac{1}{N} \sum_{\ell=k+1}^K \frac{h_\ell^2 p_\ell h_k^2 p_k}{h_k^2 p_k + h_\ell^2 p_\ell \gamma_k}} \quad (12)$$

Now, the above expression can be used to derive a simple and effective power control algorithm. Let us consider the  $K$ -th user first, i.e. the one with the smallest channel gain. The information symbol from this user will be the last one to be detected, thus implying that, under the assumption of error-free detection of previous bits, and denoting by  $P_K$  the received power for this user, the relation  $P_K/\sigma^2 = \bar{\gamma}$  should hold. As a consequence, the transmit power for the  $K$ -th user is set to  $p_K = \min\{\bar{\gamma}\sigma^2/h_K^2, P_{\max}\}$ .

Consider now user  $K-1$ ; on denoting by  $P_{K-1}$  its received power, we have that

$$\frac{P_{K-1}}{\sigma^2 + \frac{1}{N} \frac{P_{K-1} p_K h_K^2}{P_{K-1} + p_K h_K^2 \bar{\gamma}}} = \bar{\gamma}. \quad (13)$$

Now, we may reasonably assume that  $h_K$  and  $h_{K-1}$  are approximately equal (recall that these are the two smallest channel gains), and thus that  $P_{K-1} \approx P_K$ . Moreover, in a distributed approach, we can substitute  $h_K$  with its estimate  $\hat{h}_K$  given by the Lemma of the previous section. As a consequence, (13) can be approximated as

$$\frac{P_{K-1}}{\sigma^2 + \frac{1}{N} \frac{P_{K-1} p_K \hat{h}_K^2}{P_{K-1} + p_K \hat{h}_K^2 \bar{\gamma}}} = \bar{\gamma}. \quad (14)$$

After solving the above equation for  $P_{K-1}$ , we can set the transmit power of the  $(K-1)$ -th user according to  $p_{K-1} = \min\{P_{K-1}/\hat{h}_{K-1}^2, P_{\max}\}$ .

In general, for the generic  $k$ -th user, we have that the following relation must hold

$$\frac{P_k}{\sigma^2 + \frac{1}{N} \sum_{\ell=k+1}^K \frac{P_k h_\ell^2 p_\ell}{P_k + h_\ell^2 p_\ell \bar{\gamma}}} = \bar{\gamma}. \quad (15)$$

Since it is reasonable to assume that  $P_k \approx P_{k+1}$ , and replacing the channel gains with their estimates, the above equation can be re-written as

$$\frac{P_k}{\sigma^2 + \frac{1}{N} \sum_{\ell=k+1}^K \frac{P_k \hat{h}_\ell^2 p_\ell}{P_{k+1} + \hat{h}_\ell^2 p_\ell \bar{\gamma}}} = \bar{\gamma}. \quad (16)$$

Solving with respect to  $P_k$  we have

$$P_k = \bar{\gamma} \sigma^2 \frac{1}{1 - \frac{\bar{\gamma}}{N} \sum_{\ell=k+1}^K \frac{\hat{h}_\ell^2 p_\ell}{P_{k+1} + \hat{h}_\ell^2 p_\ell \bar{\gamma}}}, \quad (17)$$

and the transmit power for the  $k$ -th user is

$$p_k = \min\{P_k/\hat{h}_k^2, P_{\max}\}. \quad (18)$$

In summary, the proposed algorithm proceeds as follows. For centralized implementation, equations (17) and (18) are

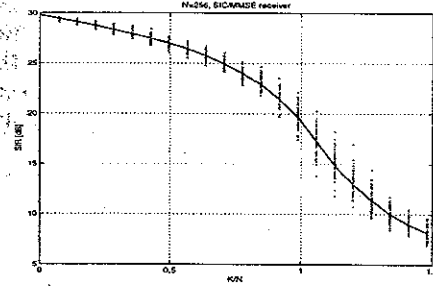


Fig. 1. Randomly generated SIC/MMSE SINR compared to the asymptotic limit, for the case of exponentially distributed powers (Rayleigh fading).

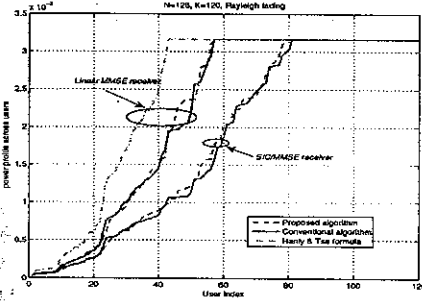


Fig. 2. Transmitted power profile across users for the proposed algorithm, the conventional power control algorithm [1] and the profile derived by the algorithm in [4], for both cases of linear MMSE detection and SIC/MMSE detection.

sequentially implemented for  $k = K, K-1, \dots, 2, 1$ . Alternatively, for decentralized implementation, the generic  $j$ -th user must compute equations (17) and the equation

$$p_k = \min\{P_k/\hat{h}_k^2, P_{\max}\}, \quad (19)$$

for  $k = K, K-1, \dots, j+1$ . Finally, equations (17) and (18) are implemented for  $k = j$ . Note that in the distributed implementation each user needs to know its own channel gain and also its order in the data detection sequence at the receiver, i.e. it must know how many users interfere with it at the receiver. However, no information on its uplink SINR or on the parameters of the multiaccess interference is needed.

## V. NUMERICAL RESULTS

We consider an uplink DS/CDMA system with processing gain  $N = 128$ , and assume that the packet length is  $M = 120$ , and that the target SINR is  $\bar{\gamma} = 6.689 = 8.25$  dB. According to [7], it can be seen that achieving the target SINR  $\bar{\gamma}$  in the considered scenario leads to a maximization of the utility, i.e. of the ratio between the packet success rate and the transmit power.

A single-cell system is considered, wherein users may have random positions with a distance from the BS ranging from 10m to 1000m. The channel coefficient  $h_k$  for the generic  $k$ -th user is assumed to be Rayleigh distributed with mean equal to  $d_k^{-1}$ , with  $d_k$  being the distance of user  $k$  from the BS. As to the thermal noise level, we take  $\sigma^2 = 2 \cdot 10^{-9}$  W/Hz, while the maximum allowed power  $P_{k,\max}$  is  $-25$  dBW. Fig. 2 reports

the transmitted power profile across users for the algorithm proposed in Section III and IV, for the algorithm derived by Eq. (16) in [4] (i.e. eq. (6)), and for a conventional algorithm (see [1]) that is non-adaptive and requires a substantial amount of prior information. It is seen that the proposed algorithms are capable of reproducing the optimal power profile with very good accuracy, while, on the contrary, the algorithm descending from paper [4] overestimates the required transmit powers and does not exhibit good performance. It is seen clearly that adopting a SIC/MMSE receiver yields considerable savings in transmit power needed to achieve a certain target SINR.

While Fig. 2 shows the result of only one simulation trial (note however that similar behavior has been observed in every case we considered), the remaining three figures report results coming from an average over 1000 independent trials. Figs. 3 - 5 show the achieved average utility (measured in bits/Joule), the average per-user transmit power and the average achieved SINR at the receiver output for the conventional power control algorithms (for both linear MMSE and SIC/MMSE detection) [1], for the proposed algorithms (for both linear MMSE and SIC/MMSE detection), and for the power control algorithm derived by paper [4]. These results show that the proposed algorithms achieve performance levels practically indistinguishable from those of the standard algorithms, while the algorithm (6) achieves a much smaller utility. From Fig. 5 it is however seen that the algorithm (6) achieves an output SINR larger than that of any other algorithm considered: this should not be interpreted as a sign of good performance. Indeed, in the considered scenario the aim of the power control algorithm is to make each user operate at a SINR equal to  $\bar{\gamma}$ .

## VI. CONCLUSIONS

This paper has considered the design of distributed power control algorithms for cellular CDMA systems based on asymptotic analysis, for situations in which either linear MMSE detection or a non-linear SIC/MMSE detection are used by the receiver. For the latter case, closed-form formulas for the limiting system SINR have also been developed.

Overall, the proposed solutions achieve satisfactory performance, and the proposed approach is quite promising. Among the authors' current research efforts in this area is the extension of the proposed algorithms to the situation in which the received signals have been affected by multipath.

## ACKNOWLEDGEMENTS

This research was supported in part by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252 and in part by the U. S. Defense Advanced Research Projects Agency under Grant HR0011-06-1-0052.

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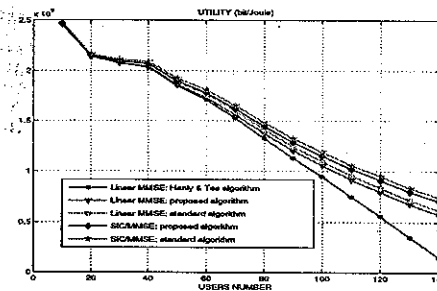


Fig. 3. Average utility versus number of users.

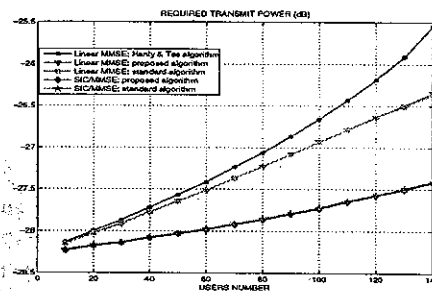


Fig. 4. Average transmit power versus number of users.

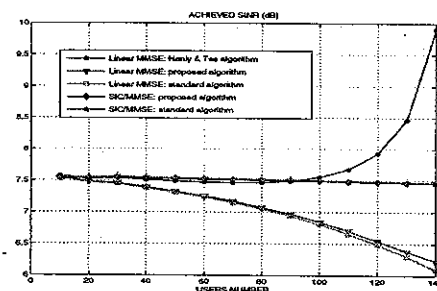


Fig. 5. Average achieved SINR versus number of users.

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# A Stochastic Non-Cooperative Game for Energy Efficiency in Wireless Data Networks

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**Summary.** In this paper the issue of energy efficiency in CDMA wireless data networks is addressed through a game theoretic approach. Building on a recent paper by the first two authors, wherein a non-cooperative game for spreading-code optimization, power control, and receiver design has been proposed to maximize the ratio of data throughput to transmit power for each active user, a stochastic algorithm is here described to perform adaptive implementation of the said non-cooperative game. The proposed solution is based on a combination of RLS-type and LMS-type adaptations, and makes use of readily available measurements. Simulation results show that its performance approaches with satisfactory accuracy that of the non-adaptive game, which requires a much larger amount of prior information.

## 1 Introduction

Game theory [1] is a branch of mathematics that has been applied primarily in economics and other social sciences to study the interactions among several autonomous subjects with contrasting interests. More recently, it has been discovered that it can also be used for the design and analysis of communication systems, mostly with application to resource allocation algorithms [2], and, in particular, to power control [3]. As examples, the reader is referred to [4, 5, 6]. Here, for a multiple access wireless data network, noncooperative and cooperative games are introduced, wherein each user chooses its transmit power in order to maximize its own utility, defined as the ratio of the throughput to transmit power. While the above papers consider the issue of power control

\* This paper was supported by the U. S. Air Force Research Laboratory under Cooperative Agreement No. FA8750-06-1-0252, and by the U. S. Defense Advanced Research Projects Agency under Grant No. HR0011-06-1-0052.

assuming that a conventional matched filter is available at the receiver, reference [7] considers the problem of joint linear receiver design and power control so as to maximize the utility of each user. It is shown here that the inclusion of receiver design in the considered game brings remarkable advantages, and, also, results based on the powerful large-system analysis are presented. More recently, the results of [7] have been extended in [8] to the case in which also each user's spreading code is included in the tunable parameters for utility maximization. The study [8] thus shows that significant performance gains can be obtained through the joint optimization of the spreading code, the transmit power and the receiver filter for each user.

On the other hand, the solutions proposed in [7] and [8], while providing a general framework for cross-layer resource optimization through a game theoretic approach, describe solutions based on a perfect knowledge of a number of parameters such as the spreading codes, the transmit powers, the propagation channels and the receive filters for all the users. Otherwise stated, the optimization procedure for each user requires a vast amount of prior information not only for the user of interest, but also for all the remaining active users. In this paper, instead, we consider the more practical and challenging situation in which each user performs utility maximization based on the knowledge of its parameters only, i.e. assuming total ignorance of the interference background. This may be the usual scenario in the downlink of a wireless data network, as well as in the uplink of a multicell wireless network, wherein each access point (AP) is disturbed by the interference originating from users served by surrounding AP's. An adaptively learning algorithm capable of approaching with good accuracy the performance of the non-adaptive game is thus presented, based on a combination of the recursive-least-squares (RLS) and least-mean-squares (LMS) adaptation rules. The proposed algorithm assumes no prior knowledge on the interference background and makes use of readily available measurements.

The rest of this paper is organized as follows. The next section contains some preliminaries and the system model of interest. Section 3 contains a brief review of the non-adaptive game considered in [8], while the stochastic implementation of the resource allocation algorithm is detailed in Section 4. Section 5 contains extensive simulation results, while, finally concluding remarks are given in Section 6.

## 2 Preliminaries and problem statement

Consider the uplink of a  $K$ -user synchronous, single-cell, direct-sequence code division multiple access (DS/CDMA) network with processing gain  $N$  and subject to flat fading. After chip-matched filtering and sampling at the chip-rate, the  $N$ -dimensional received data vector, say  $\mathbf{r}$ , corresponding to one symbol interval, can be written as

$$\mathbf{r} = \sum_{k=1}^K \sqrt{p_k} h_k b_k \mathbf{s}_k + \mathbf{n}, \quad (1)$$

wherein  $p_k$  is the transmit power of the  $k$ -th user<sup>3</sup>,  $b_k \in \{-1, 1\}$  is the information symbol of the  $k$ -th user, and  $h_k$  is the real<sup>4</sup> channel gain between the  $k$ -th user's transmitter and the access point (AP); the actual value of  $h_k$  depends on both the distance of the  $k$ -th user's terminal from the AP and the channel fading fluctuations. The  $N$ -dimensional vector  $\mathbf{s}_k$  is the spreading code of the  $k$ -th user; we assume that the entries of  $\mathbf{s}_k$  are real and that  $\mathbf{s}_k^T \mathbf{s}_k = \|\mathbf{s}_k\|^2 = 1$ , with  $(\cdot)^T$  denoting transpose. Finally,  $\mathbf{n}$  is the ambient noise vector, which we assume to be a zero-mean white Gaussian random process with covariance matrix  $(N_0/2)\mathbf{I}_N$ , with  $\mathbf{I}_N$  the identity matrix of order  $N$ . An alternative and compact representation of (1) is given by

$$\mathbf{r} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{H}\mathbf{b} + \mathbf{n}, \quad (2)$$

wherein  $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_K]$  is the  $N \times K$ -dimensional spreading code matrix,  $\mathbf{P}$  and  $\mathbf{H}$  are  $K \times K$ -dimensional diagonal matrices, whose diagonals are  $[p_1, \dots, p_K]$  and  $[h_1, \dots, h_K]$ , respectively, and; finally,  $\mathbf{b} = [b_1, \dots, b_K]^T$  is the  $K$ -dimensional vector of the data symbols.

Assume now that each mobile terminal sends its data in packets of  $M$  bits, and that it is interested both in having its data received with as small as possible error probability at the AP, and in making careful use of the energy stored in its battery. Obviously, these are conflicting goals, since error-free reception may be achieved by increasing the received SNR, i.e. by increasing the transmit power, which of course comes at the expense of battery life<sup>5</sup>. A useful approach to quantify these conflicting goals is to define the utility of the  $k$ -th user as the ratio of its throughput, defined as the number of information bits that are received with no error in unit time, to its transmit power [4, 5], i.e.

$$u_k = \frac{T_k}{p_k}. \quad (3)$$

Note that  $u_k$  is measured in bit/Joule, i.e. it represents the number of successful bit transmissions that can be made for each Joule of energy drained from the battery. Denoting by  $R$  the common rate of the network (extension

<sup>3</sup> To simplify subsequent notation, we assume that the transmitted power  $p_k$  subsumes also the gain of the transmit and receive antennas.

<sup>4</sup> We assume here, for simplicity, a real channel model; generalization to practical channels, with I and Q components, is straightforward.

<sup>5</sup> Of course there are many other strategies to lower the data error probability, such as for example the use of error correcting codes, diversity exploitation, and implementation of optimal reception techniques at the receiver. Here, however, we are mainly interested to energy efficient data transmission and power usage, so we consider only the effects of varying the transmit power, the receiver and the spreading code on energy efficiency.

to the case in which each user transmits with its own rate  $R_k$  is quite simple) and assuming that each packet of  $M$  symbols contains  $L$  information symbols and  $M - L$  overhead symbols, reserved, e.g., for channel estimation and/or parity checks, the throughput  $T_k$  can be expressed as

$$T_k = R \frac{L}{M} P_k \quad (4)$$

wherein  $P_k$  denotes the probability that a packet from the  $k$ -th user is received error-free. In the considered DS/CDMA setting, the term  $P_k$  depends formally on a number of parameters such as the spreading codes of all the users and the diagonal entries of the matrices  $\mathbf{P}$  and  $\mathbf{H}$ , as well as on the strength of the used error correcting codes. However, a customary approach is to model the multiple access interference as a Gaussian random process, and assume that  $P_k$  is an increasing function of the  $k$ -th user's Signal-to-Interference plus Noise-Ratio (SINR)  $\gamma_k$ , which is naturally the case in many practical situations.

Recall that, for the case in which a linear receiver is used to detect the data symbol  $b_k$ , according, i.e., to the decision rule

$$\hat{b}_k = \text{sign} \left[ \mathbf{d}_k^T \mathbf{r} \right], \quad (5)$$

with  $\hat{b}_k$  the estimate of  $b_k$  and  $\mathbf{d}_k$  the  $N$ -dimensional vector representing the receive filter for the user  $k$ , it is easily seen that the SINR  $\gamma_k$  can be written as

$$\gamma_k = \frac{p_k h_k^2 (\mathbf{d}_k^T \mathbf{s}_k)^2}{\frac{N_0}{2} \|\mathbf{d}_k\|^2 + \sum_{i \neq k} p_i h_i^2 (\mathbf{d}_k^T \mathbf{s}_i)^2} \quad (6)$$

Of related interest is also the mean square error (MSE) for the user  $k$ , which, for a linear receiver, is defined as

$$\text{MSE}_k = E \left\{ \left( b_k - \mathbf{d}_k^T \mathbf{r} \right)^2 \right\} = 1 + \mathbf{d}_k^T \mathbf{M} \mathbf{d}_k - 2\sqrt{p_k} h_k \mathbf{d}_k^T \mathbf{s}_k, \quad (7)$$

wherein  $E\{\cdot\}$  denotes statistical expectation and  $\mathbf{M} = \left( \mathbf{S} \mathbf{H} \mathbf{P} \mathbf{H}^T \mathbf{S}^T + \frac{N_0}{2} \mathbf{I}_N \right)$  is the covariance matrix of the data.

The exact shape of  $P_k(\gamma_k)$  depends on factors such as the modulation and coding type. However, in all cases of relevant interest, it is an increasing function of  $\gamma_k$  with a sigmoidal shape, and converges to unity as  $\gamma_k \rightarrow +\infty$ ; as an example, for binary phase-shift-keying (BPSK) modulation coupled with no channel coding, it is easily shown that

$$P_k(\gamma_k) = \left[ 1 - Q(\sqrt{2\gamma_k}) \right]^M, \quad (8)$$

with  $Q(\cdot)$  the complementary cumulative distribution function of a zero-mean random Gaussian variate with unit variance. A plot of (8) is shown in Fig. 1 for the case  $M = 100$ .

It should be also noted that substituting (8) into (4), and, in turn, into (3), leads to a strong incongruence. Indeed, for  $p_k \rightarrow 0$ , we have  $\gamma_k \rightarrow 0$ , but  $P_k$  converges to a small but non-zero value (i.e.  $2^{-M}$ ), thus implying that an unboundedly large utility can be achieved by transmitting with zero power, i.e. not transmitting at all and making blind guesses at the receiver on what data were transmitted. To circumvent this problem, a customary approach [5, 7] is to replace  $P_k$  with an *efficiency function*, say  $f_k(\gamma_k)$ , whose behavior should approximate as close as possible that of  $P_k$ , except that for  $\gamma_k \rightarrow 0$  it is required that  $f_k(\gamma_k) = o(\gamma_k)$ . The function  $f(\gamma_k) = (1 - e^{-\gamma_k})^M$  is a widely accepted substitute for the true probability of correct packet reception, and in the following we will adopt this model<sup>6</sup>. This efficiency function is increasing and S-shaped, converges to unity as  $\gamma_k$  approaches infinity, and has a continuous first order derivative. Note that we have omitted the subscript “ $k$ ”, i.e. we have used the notation  $f(\gamma_k)$  in place of  $f_k(\gamma_k)$  since we assume that the efficiency function is the same for all the users.

Summing up, substituting (4) into (3) and replacing the probability  $P_k$  with the above defined efficiency function, we obtain the following expression for the  $k$ -th user's utility:

$$u_k = R \frac{L}{M} \frac{f(\gamma_k)}{p_k}, \quad \forall k = 1, \dots, K. \quad (9)$$

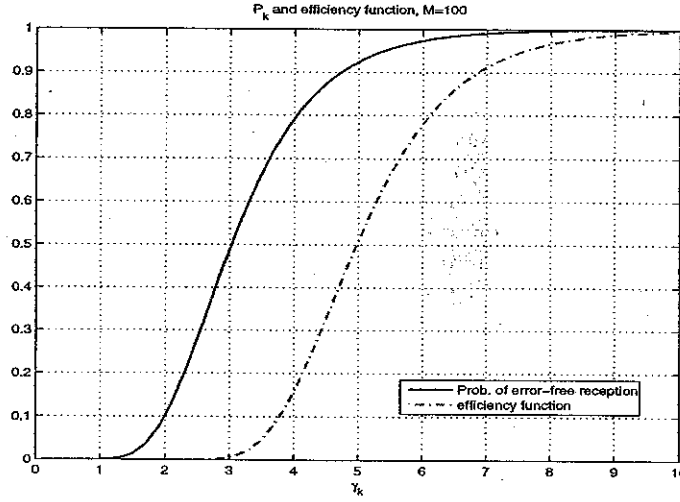


Fig. 1. Comparison of probability of error-free packet reception and efficiency function versus receive SINR and for packet size  $M = 100$ . Note the S-shape of both functions.

<sup>6</sup> See Fig. 1 for a comparison between the Probability  $P_k$  and the efficiency function.



### 3 The non-cooperative game with linear receivers

Based on the utility definition (9), it is natural to wonder how each user may maximize its utility, how this maximization affects utilities achieved by other users, and, also, if a stable equilibrium point does exist. These questions have been answered in recent papers by resorting to the tools of game theory. As an example, non-cooperative scenarios wherein mobile users are allowed to vary their transmit power only have been considered in [4, 5, 6]; in [7], instead, such an approach has been extended to the cross layer scenario in which each user may vary its power and its uplink linear receiver, while, more recently, reference [8] has considered the case in which each user is able to tune the transmit power, the uplink linear receiver and the adopted spreading code.

Formally, the game  $\mathcal{G}$  considered in [8] can be described as the triplet  $\mathcal{G} = [\mathcal{K}, \{S_k\}, \{u_k\}]$ , wherein  $\mathcal{K} = \{1, 2, \dots, K\}$  is the set of active users participating in the game,  $u_k$  is the  $k$ -th user's utility defined in (9), and

$$S_k = [0, P_{k,\max}] \times \mathcal{R}^N \times \mathcal{R}_1^N, \quad (10)$$

is the set of possible actions (strategies) that user  $k$  can take. It is seen that  $S_k$  is written as the Cartesian product of three different sets, and indeed  $[0, P_{k,\max}]$  is the range of available transmit powers for the  $k$ -th user (note that  $P_{k,\max}$  is the maximum allowed transmit power for user  $k$ ),  $\mathcal{R}^N$ , with  $\mathcal{R}$  the real line, defines the set of all possible linear receive filters, and, finally,

$$\mathcal{R}_1^N = \{d \in \mathcal{R}^N : d^T d = 1\},$$

defines the set of the allowed spreading codes<sup>7</sup> for user  $k$ .

Summing up, the proposed noncooperative game can be cast as the following maximization problem

$$\max_{S_k} u_k = \max_{p_k, d_k, s_k} u_k(p_k, d_k, s_k), \quad \forall k = 1, \dots, K. \quad (11)$$

Given (9), the above maximization can be also written as

$$\max_{p_k, d_k, s_k} \frac{f(\gamma_k(p_k, d_k, s_k))}{p_k}, \quad \forall k = 1, \dots, K. \quad (12)$$

Moreover, since the efficiency function is monotone and non-decreasing, we also have

$$\max_{p_k, d_k, s_k} \frac{f(\gamma_k(p_k, d_k, s_k))}{p_k} = \max_{p_k} \frac{f\left(\max_{d_k, s_k} \gamma_k(p_k, d_k, s_k)\right)}{p_k}, \quad (13)$$

<sup>7</sup> Here we assume that the spreading codes have real entries; the problem of utility maximization with reasonable complexity for the case of discrete-valued entries is a challenging issue that will be considered in the future.

i.e. we can first take care of SINR maximization with respect to spreading codes and linear receivers, and then focus on maximization of the resulting utility with respect to transmit power.

Letting  $(\cdot)^+$  denoting Moore-Penrose pseudoinverse, the following result is reported in [8].

**Proposition:** *The non-cooperative game defined in (11) admits a unique Nash equilibrium point  $(p_k^*, d_k^*, s_k^*)$ , for  $k = 1, \dots, K$ , wherein*

- $s_k^*$  and  $d_k^*$  are the unique fixed stable  $k$ -th user spreading code and receive filter<sup>8</sup> resulting from iterations

$$\begin{aligned} d_i &= \sqrt{p_i} h_i \left( S H P H^T S^T + \frac{N_0}{2} I_N \right)^{-1} s_i \quad \forall i = 1, \dots, K \\ s_i &= \sqrt{p_i} h_i \left( p_i h_i^2 D D^T + \mu_i I_N \right)^+ d_i \quad \forall i = 1, \dots, K \end{aligned} \quad (14)$$

with  $\mu_i$  such that  $\|s_i\|^2 = 1$ , and  $D = [s_1, \dots, s_K]$ . Denote by  $\gamma_k^*$  the corresponding SINR.

- $p_k^* = \min\{\bar{p}_k, P_{k,\max}\}$ , with  $\bar{p}_k$  the  $k$ -th user transmit power such that the  $k$ -th user maximum SINR  $\gamma_k^*$  equals  $\bar{\gamma}$ , i.e. the unique solution of the equation  $f(\gamma) = \gamma f'(\gamma)$ , with  $f'(\gamma)$  the derivative of  $f(\gamma)$ .

In practice, this result states that the non-cooperative game (11) admits a unique Nash equilibrium, that can be reached as follows. First of all, the equation  $f(\gamma) = \gamma f'(\gamma)$  is to be solved in order to determine its unique solution  $\bar{\gamma}$ ; then, an iterative procedure starts wherein the system alternates between these two phases:

- a. Given the transmit powers, each user adjusts its spreading code and receive filter through iterations (14) until an equilibrium is reached;
- b. Given the spreading codes and uplink receivers, each user tunes its transmit power so that its own SIR equals  $\bar{\gamma}$ . Denoting by  $\mathbf{p} = [p_1, \dots, p_K]$  the users' power vector, and by  $\mathbf{I}(\mathbf{p})$  the  $K$ -dimensional vector whose  $k$ -th entry  $I_k(\mathbf{p})$  is written as

$$I_k(\mathbf{p}) = \frac{\bar{\gamma}}{h_k^2 (d_k s_k)^2} \left( \frac{N_0}{2} \|d_k\|^2 + \sum_{i \neq k} p_i h_i^2 (d_k s_i)^2 \right), \quad (15)$$

the transmit power vector  $\mathbf{p}$  is the unique fixed stable point of the iteration [3]

$$p_k = \begin{cases} I_k(\mathbf{p}), & \text{for } I_k(\mathbf{p}) \leq P_{k,\max}, \\ P_{k,\max}, & \text{for } I_k(\mathbf{p}) > P_{k,\max}, \end{cases} \quad (16)$$

for all  $k = 1, \dots, K$ .

<sup>8</sup> Actually the linear receive filter is unique up to a positive scaling factor.

Steps a. and b. are to be repeated until convergence is reached. It is crucial to note that computation of the equilibrium transmit power, spreading code and linear receiver for each user needs a lot of prior information. In particular, it is seen from eq. (14) that computation of the  $k$ -th user receiver requires knowledge of the spreading codes, transmit powers and channel gains for all the active users, while computation of the  $k$ -th user spreading code requires knowledge of  $\mathbf{D}$ , i.e. the matrix of the uplink receivers for all the users. Likewise, implementation of iterations (16) also requires the same vast amount of prior information. Our next goal is thus to propose a stochastic resource allocation algorithm that alleviates the need for such prior knowledge, and that is amenable to a decentralized implementation, wherein each user may allocate its own resources based only on knowledge that is readily available, and with total ignorance on the interference background.

#### 4 Adaptive energy efficient resource allocation

- In order to illustrate the adaptive implementation of the non-cooperative game, a slight change of notation is needed. Indeed, since any adaptive algorithm relies on several data observations in consecutive symbol intervals, we cannot restrict any longer our attention to one symbol interval only, and we thus denote by  $\mathbf{r}(n)$  the  $N$ -dimensional received data vector in the  $n$ -th bit interval, i.e.

$$\mathbf{r}(n) = \sum_{k=1}^K \sqrt{p_k(n)} h_k \mathbf{b}_k(n) s_k(n) + \mathbf{w}(n). \quad (17)$$

Eq. (17) differs from Eq. (1) in that a temporal index has been added to some parameters, to underline their time-varying nature: as an example,  $p_k(n)$  and  $s_k(n)$  are the transmit power and the spreading code of the  $k$ -th user in the  $n$ -th symbol interval. Note also that the channel gain does not depend on  $n$ , i.e. we are implicitly assuming a slow fading channel, even though generalization to the case of slowly time-varying channels is quite straightforward. In order to obtain an adaptive implementation of the utility maximizing algorithm for the generic  $k$ -th user, first we focus on iterations (14); as specified in [8, 9], the unique fixed stable point of these iterations achieves the global minimum for the total mean square error (TMSE), which is given by

$$\text{TMSE} = \sum_{k=1}^K \text{MSE}_k, \quad (18)$$

with  $\text{MSE}_k$  defined as in (7). It can be shown, indeed, that minimization of the TMSE leads to a Pareto-optimal solution to the problem of maximizing the SINR for each user. On the other hand, an alternative approach is to consider the case in which each user tries to minimize its own MSE with respect to its

spreading code and linear receiver. The  $k$ -th user MSE can be shown to be written as

$$\text{MSE}_k = 1 + \mathbf{d}_k^T (p_k h_k^2 \mathbf{s}_k \mathbf{s}_k^T + \mathbf{M}_k) \mathbf{d}_k - 2\sqrt{p_k} h_k \mathbf{d}_k^T \mathbf{s}_k, \quad (19)$$

with  $\mathbf{M}_k = \mathbf{M} - p_k h_k^2 \mathbf{s}_k \mathbf{s}_k^T$ . Minimization of Eq. (19) with respect to  $\mathbf{d}_k$  and  $\mathbf{s}_k$ , under the constraint  $\|\mathbf{s}_k\|^2 = 1$ , yields the iterations

$$\begin{aligned} \mathbf{d}_i &= \sqrt{p_i} h_i \left( \mathbf{S} \mathbf{H} \mathbf{P} \mathbf{H}^T \mathbf{S}^T + \frac{N_0}{2} \mathbf{I}_N \right)^{-1} \mathbf{s}_i \quad \forall i = 1, \dots, K \\ \mathbf{s}_i &= \sqrt{p_i} h_i \left( p_i h_i^2 \mathbf{d}_i \mathbf{d}_i^T + \mu_i \mathbf{I}_N \right)^{-1} \mathbf{d}_i \quad \forall i = 1, \dots, K \end{aligned} \quad (20)$$

In general, minimization of the TMSE is not equivalent to individual minimization of the MSE of each user; however, in our scenario, i.e. in the case of a single-path fading channel, the two approaches can be shown to be equivalent [10]. As a consequence, we can state that the fixed point of iterations (14) coincides with that of iterations (20). Note however that, despite such equivalence, the spreading code update for each user in (20) depends only on parameters of the user itself, and does not require any knowledge on the interference background. The receiver updates in (14) and (20) are the same, and indeed they coincide with the MMSE receiver. Accordingly, since the utility maximizing linear receiver is the MMSE filter, we start resorting to the well-known recursive-least-squares (RLS) implementation of this receiver. Letting  $\mathbf{R}(0) = \epsilon \mathbf{I}_N$ , with  $\epsilon$  a small positive constant, letting  $\lambda$  be a close-to-unity scalar constant, assuming that the receiver has knowledge of the information symbols  $b_k(1), \dots, b_k(T)$ , and denoting by  $\mathbf{d}_k(n)$  the estimate of the linear receiver filter for the  $k$ -th user in the  $n$ -th symbol interval, the following iterations can be considered

$$\begin{aligned} \mathbf{k}(n) &= \frac{\mathbf{R}^{-1}(n-1) \mathbf{r}(n)}{\lambda + \mathbf{r}^T(n) \mathbf{R}^{-1}(n-1) \mathbf{r}(n)}, \\ \mathbf{R}^{-1}(n) &= \frac{1}{\lambda} [\mathbf{R}^{-1}(n-1) - \mathbf{k}(n) \mathbf{r}^T(n) \mathbf{R}^{-1}(n-1)], \\ \mathbf{e}_k(n) &= \mathbf{d}_k^T(n-1) \mathbf{r}(n) - b_k(n), \\ \mathbf{d}_k(n) &= \mathbf{d}_k(n-1) - \mathbf{e}_k(n) \mathbf{k}(n). \end{aligned} \quad (21)$$

The last line in (21) represents the update equation for the detection vector  $\mathbf{d}_k(\cdot)$ . Note that this equation, in turn, depends on the error vector  $\mathbf{e}_k(n)$ , which, for  $n \leq T$ , can be built based on the knowledge of the training symbol  $b_k(n)$ . Once the training phase is over, real data detection takes place and the error in the third line of (21) is computed according to the equation

$$\mathbf{e}_k(n) = \mathbf{d}_k^T(n-1) \mathbf{r}(n) - \text{sgn} [\mathbf{d}_k^T(n-1) \mathbf{r}(n)]. \quad (22)$$

Given its own receive filter  $\mathbf{d}_k(n)$ , the  $k$ -th user can then modify its spreading code according to the second line of eq. (20), i.e.:

$$s_k(n+1) = \sqrt{p_k(n)h_k} \left( p_k(n)h_k^2 d_k(n) d_k^T(n) + \mu_k(n) I_N \right)^{-1} d_k(n), \quad (23)$$

with  $\mu_k(n)$  a constant such that  $s_k^T(n)s_k(n) = 1$ . Note that the update in (23) only requires parameters of the  $k$ -th user, thus implying that no knowledge on the interference background is needed. Finally, we have to consider the tuning of the transmit power so that each user may achieve its target SINR  $\bar{\gamma}$ . This is a classical stochastic power control problem that has been treated, for instance, in [11]. A possible solution is to consider a least-mean-squares (LMS) update of the transmit power according to the rule

$$\begin{aligned} p_k(n+1) &= (1-\rho)p_k(n) + \rho I_k(n), \\ p_k(n+1) &= \min(p_k(n+1), P_{k,\max}). \end{aligned} \quad (24)$$

In the above equations, the step-size  $\rho$  is a close-to-zero positive constant and  $I_k(n)$  is a stochastic approximation of the  $k$ -th entry of the vector  $\mathbf{I}(\mathbf{p})$ , and is expressed as

$$I_k(n) = \frac{\bar{\gamma}}{h_k(d_k^T(n)s_k(n))^2} \left[ (d_k^T(n)r(n))^2 - p_k(n)h_k^2(d_k^T(n)s_k(n))^2 \right] \quad (25)$$

Note that also the update (24) does not require any knowledge on the interference. To summarize, the algorithm proceeds as follows: for  $n \leq T$ , only the RLS update (21) is performed; then, for each  $n > T$ , the algorithm performs the updates in (21), (23) and, finally, (24). In particular, note that the power update is made in parallel to the spreading code update and receiver update, i.e. without waiting for convergence of the RLS-based adaptive implementation of the MMSE receiver.

Although a theoretical convergence study of this algorithm is definitely worth being undertaken, it is out of the scope of this paper; in the next section, we will discuss the results of extensive computer simulations that will show the excellent behavior of the outlined procedure.

## 5 Numerical Results

We contrast here the performance of the non-adaptive game discussed in [8] with its adaptive implementation proposed here. We consider an uplink DS/CDMA system with processing gain  $N = 15$ , and assume that the packet length is  $M = 120$ . For this value of  $M$  the equation  $f(\gamma) = \gamma f'(\gamma)$  can be shown to admit the solution  $\bar{\gamma} = 6.689 = 8.25\text{dB}$ . A single-cell system is considered, wherein users may have random positions with a distance from the AP ranging from 10m to 500m. The channel coefficient  $h_k$  for the generic  $k$ -th user is assumed to be Rayleigh distributed with mean equal to  $d_k^{-2}$ , with  $d_k$

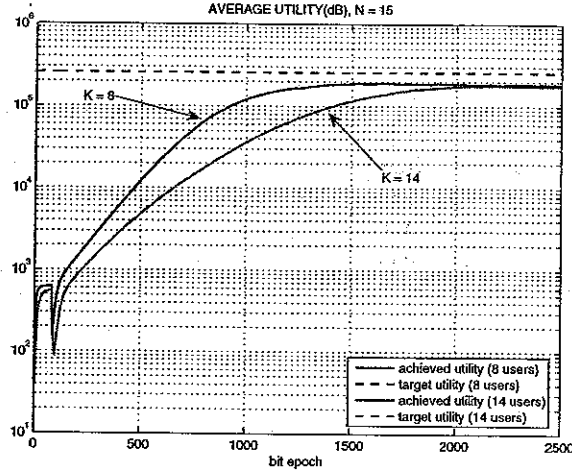


Fig. 2. Achieved average utility versus time.

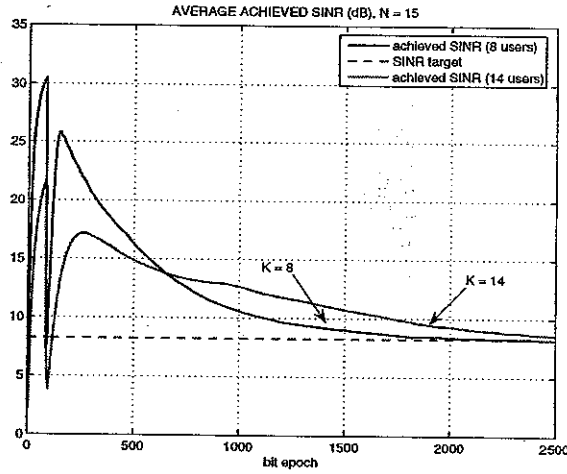


Fig. 3. Achieved average SINR versus time.

being the distance of user  $k$  from the AP<sup>9</sup>. We take the ambient noise level to be  $\mathcal{N}_0 = 10^{-5}$  W/Hz, while the maximum allowed power  $P_{k,\max}$  is 25dB. We present the results of averaging over 1000 independent realizations for the

<sup>9</sup> Note that we are here assuming that the power path losses are proportional to the fourth power of the path length, which is reasonable in urban cellular environments.

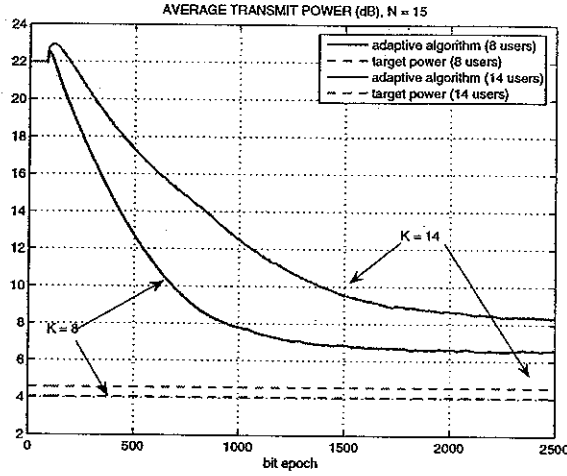


Fig. 4. Average transmit power versus time.

users locations, fading channel coefficients and starting set of spreading codes. More precisely, for each iteration we randomly generate an  $N \times K$ -dimensional spreading code matrix with entries in the set  $\{-1/\sqrt{N}, 1/\sqrt{N}\}$ ; this matrix is then used as the starting point for the considered games. We consider the case in which  $T = 80$  training symbols are used, while in eq. (24) the step size  $\rho = .01$  has been taken. Figs. 2 - 4 report the time-evolution of the achieved average utility (measured in bit/Joule), the average achieved SINR and the average transmit power, respectively, for both the cases in which  $K = 8$  and  $K = 14$ . It is seen that after about one thousand iterations the adaptive algorithm approximate with satisfactory accuracy the benchmark scenario that a non-adaptive game is played as in [8]. In particular, while the target SINR and the achieved utility are quite close to their target values, it is seen from Fig. 4 that the average transmit power is about 3dB larger than in the non-adaptive case; such a loss is not at all surprising, since it is well-known that adaptive algorithms have a steady-state error, and that their performance may only approach that of their non-adaptive counterparts.

In order to test the tracking properties of the proposed algorithm, we also consider a dynamic scenario with an initial number of users  $K = 8$ , and with two additional users entering the channel at time epochs  $n = 1000$  and  $n = 1700$ . The results are reported in Figs 5 - 7. Results clearly show that the algorithm is capable of coping with changes in the interference background.

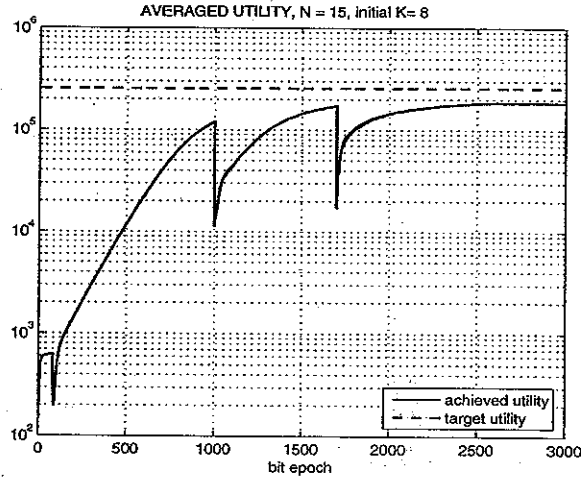


Fig. 5. Dynamic environment: Average achieved utility versus time.

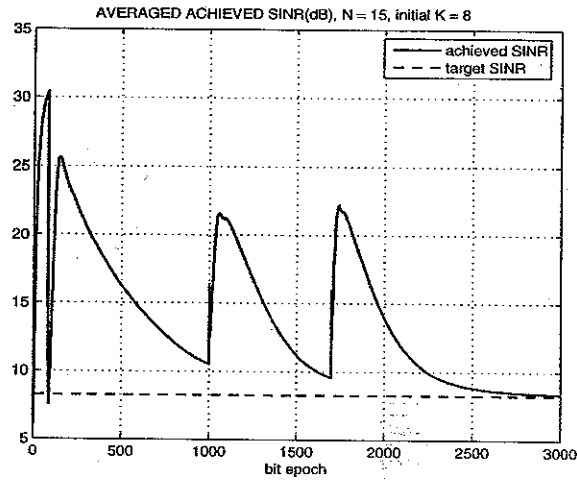


Fig. 6. Dynamic environment: Average achieved SINR versus time.

## 6 Conclusion

In this paper the cross-layer issue of joint stochastic power control, spreading code optimization and receiver design for wireless data networks has been addressed using a game-theoretic framework. Building on [8], wherein a non-cooperative game for resource allocation has been proposed, we have here



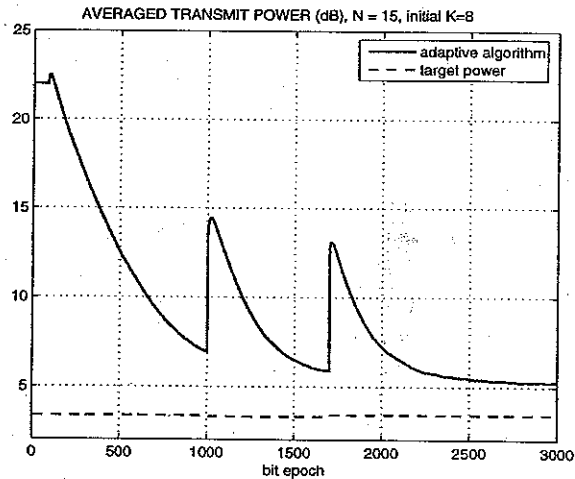


Fig. 7. Dynamic environment: Average transmit power versus time.

considered the issue of adaptive implementation of the resource allocation algorithm, based on readily available measurements and assuming no prior knowledge on the interference background. The result is thus a stochastic algorithm that can be realized in a decentralized fashion, wherein each user just needs knowledge of its own parameters. The performance of the proposed scheme has been validated through computer simulations, which showed that the adaptive implementation achieved a performance quite close to that of the non-adaptive benchmark. As interesting research topics worth being investigated we mention the theoretical analysis of the convergence properties of the algorithm, and the development and the analysis of adaptive algorithms able to implement the said game without prior knowledge of the fading channel coefficient of the user of interest.

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